ON FLOATING BODIES.

BOOK I.

Postulate 1.

"Let it be supposed that a fluid is of such a character that, its parts lying evenly and being continuous, that part which is thrust the less is driven along by that which is thrust the more; and that each of its parts is thrust by the fluid which is above it in a perpendicular direction if the fluid be sunk in anything and compressed by anything else."

Proposition 1.

If a surface be cut by a plane always passing through a certain point, and if the section be always a circumference [*of a circle*] *whose centre is the aforesaid point, the surface is that of a sphere.*

For, if not, there will be some two lines drawn from the point to the surface which are not equal.

Suppose O to be the fixed point, and A, B to be two points on the surface such that OA, OB are unequal. Let the surface be cut by a plane passing through OA, OB. Then the section is, by hypothesis, a circle whose centre is O.

Thus $OA = OB$; which is contrary to the assumption. Therefore the surface cannot but be a sphere.

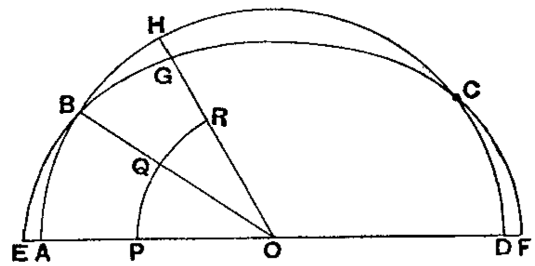
Proposition 2.

The surface of any fluid at rest is the surface of a sphere whose centre is the same as that of the earth.

Suppose the surface of the fluid cut by a plane through O, the centre of the earth, in the curve $ABCD$.

$ABCD$ shall be the circumference of a circle.

For, if not, some of the lines drawn from O to the curve will be unequal. Take one of them, OB, such that OB is greater than some of the lines from O to the curve and less than others. Draw a circle with OB as radius. Let it be EBF, which will therefore fall partly within and partly without the surface of the fluid.



Draw OGH making with OB an angle equal to the angle EOB, and meeting the surface in H and the circle in G. Draw also in the plane an arc of a circle PQR with centre O and within the fluid.

Then the parts of the fluid along PQR are uniform and continuous, and the part PQ is compressed by the part between it and AB, while the part QR is compressed by the part between QR and BH. Therefore the parts along PQ, QR will be unequally compressed, and the part which is compressed the less will be set in motion by that which is compressed the more.

Therefore there will not be rest; which is contrary to the hypothesis.

Hence the section of the surface will be the circumference of a circle whose centre is O; and so will all other sections by planes through O.

Therefore the surface is that of a sphere with centre O.

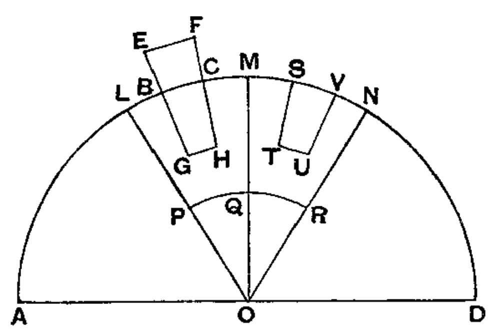
Proposition 3.

Of solids those which, size for size, are of equal weight with a fluid will, if let down into the fluid, be immersed so that they do not project above the surface but do not sink lower.

If possible, let a certain solid *EFHG* of equal weight, volume for volume, with the fluid remain immersed in it so that part of it, *EBCF*, projects above the surface.

Draw through *O*, the centre of the earth, and through the solid a plane cutting the surface of the fluid in the circle *ABCD*.

Conceive a pyramid with vertex *O* and base a parallelogram at the surface of the fluid, such that it includes the immersed portion of the solid. Let this pyramid be cut by the plane of *ABCD* in *OL*, *OM*. Also let a sphere within the fluid and below *GH* be described with centre *O*, and let the plane of *ABCD* cut this sphere in *PQR*.



Conceive also another pyramid in the fluid with vertex *O*, continuous with the former pyramid and equal and similar to it. Let the pyramid so described be cut in *OM*, *ON* by the plane of *ABCD*.

Lastly, let *STUV* be a part of the fluid within the second pyramid equal and similar to the part *BGHC* of the solid, and let *SV* be at the surface of the fluid.

Then the pressures on *PQ*, *QR* are unequal, that on *PQ* being the greater. Hence the part at *QR* will be set in motion by that at PQ, and the fluid will not be at rest; which is contrary to the hypothesis.

Therefore the solid will not stand out above the surface.

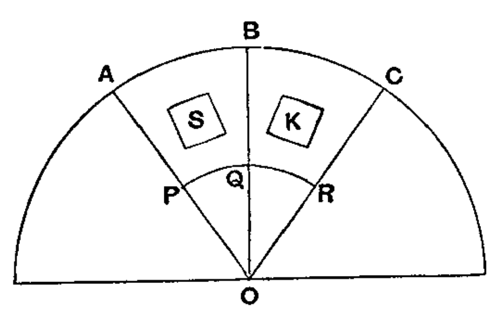
Nor will it sink further, because all the parts of the fluid will be under the same pressure.

Proposition 4.

A solid lighter than a fluid will, if immersed in it, not be completely submerged, but part of it will project above the surface.

In this case, after the manner of the previous proposition, we assume the solid, if possible, to be completely submerged and the fluid to be at rest in that position, and we conceive (1) a pyramid with its vertex at O, the centre of the earth, including the solid, (2) another pyramid continuous with the former and equal and similar to it, with the same vertex O, (3) a portion of the fluid within this latter pyramid equal to the immersed solid in the other pyramid, (4) a sphere with centre O whose surface is below the immersed solid and the part of the fluid in the second pyramid corresponding thereto. We suppose a plane to be drawn through the centre O cutting the surface of the fluid in the circle ABC, the solid in S, the first pyramid in OA, OB, the second pyramid in OB, OC, the portion of the fluid in the second pyramid in K, and the inner sphere in PQR.

Then the pressures on the parts of the fluid at PQ, QR are unequal, since S is lighter than K. Hence there will not be rest; which is contrary to the hypothesis.

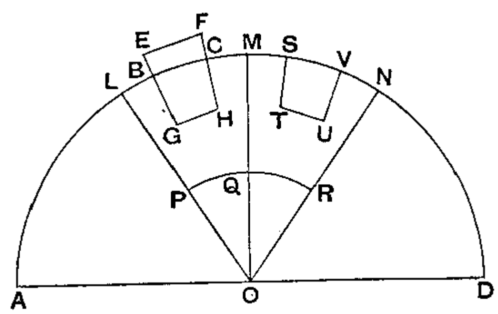


Therefore the solid S cannot, in a condition of rest, be completely submerged.

Proposition 5.

Any solid lighter than a fluid will, if placed in the fluid, be so far immersed that the weight of the solid will be equal to the weight of the fluid displaced.

For let the solid be $EGHF$, and let $BGHC$ be the portion of it immersed when the fluid is at rest. As in Prop. 3, conceive a pyramid with vertex O including the solid, and another pyramid with the same vertex continuous with the former and equal and similar to it. Suppose a portion of the fluid $STUV$ at the base of the second pyramid to be equal and similar to the immersed portion of the solid; and let the construction be the same as in Prop. 3.



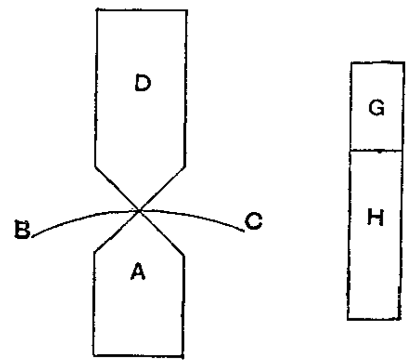
Then, since the pressure on the parts of the fluid at PQ, QR must be equal in order that the fluid may be at rest, it follows that the weight of the portion $STUV$ of the fluid must be equal to the weight of the solid $EGHF$. And the former is equal to the weight of the fluid displaced by the immersed portion of the solid $BGHC$.

Proposition 6.

If a solid lighter than a fluid be forcibly immersed in it, the solid will be driven upwards by a force equal to the difference between its weight and the weight of the fluid displaced.

For let A be completely immersed in the fluid, and let G represent the weight of A, and $(G+H)$ the weight of an equal volume of the fluid. Take a solid D, whose weight is H

and add it to A. Then the weight of $(A+D)$ is less than that of an equal volume of the fluid; and, if $(A+D)$ is immersed in the fluid, it will project so that its weight will be equal to the weight of the fluid displaced. But its weight is $(G+H)$.



Therefore the weight of the fluid displaced is $(G+H)$, and hence the volume of the fluid displaced is the volume of the solid A. There will accordingly be rest with A immersed and D projecting.

Thus the weight of D balances the upward force exerted by the fluid on A, and therefore the latter force is equal to H, which is the difference between the weight of A and the weight of the fluid which A displaces.

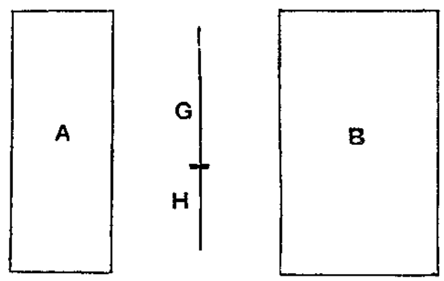
Proposition 7.

A solid heavier than a fluid will, if placed in it, descend to the bottom of the fluid, and the solid will, when weighed in the fluid, be lighter than its true weight by the weight of the fluid displaced.

(1) The first part of the proposition is obvious, since the part of the fluid under the solid will be under greater pressure, and therefore the other parts will give way until the solid reaches the bottom.

(2) Let A be a solid heavier than the same volume of the fluid, and let $(G+H)$ represent its weight, while G represents the weight of the same volume of the fluid.

Take a solid B lighter than the same volume of the fluid, and such that the weight of B is G, while the weight of the same volume of the fluid is $(G+H)$.



Let A and B be now combined into one solid and immersed. Then, since $(A+B)$ will be of the same weight as the same volume of fluid, both weights being equal to $(G+H)+G$, it follows that $(A+B)$ will remain stationary in the fluid.

Therefore the force which causes A by itself to sink must be equal to the upward force exerted by the fluid on B by itself. This latter is equal to the difference between $(G+H)$ and G [Prop. 6]. Hence A is depressed by a force equal to H, i.e. its weight in the fluid is H, or the difference between $(G+H)$ and G.

[This proposition may, I think, safely be regarded as decisive of the question how Archimedes determined the proportions of gold and silver contained in the famous crown (cf. Introduction, Chapter I.). The proposition suggests in fact the following method.

Let W represent the weight of the crown, w_1 and w_2 the weights of the gold and silver in it respectively, so that $W=w_1+w_2$.

(1) Take a weight W of pure gold and weigh it in a fluid. The apparent loss of weight is then equal to the weight of the fluid displaced. If F_1 denote this weight, F_1 is thus known as the result of the operation of weighing.

It follows that the weight of fluid displaced by a weight w_1 of gold is $\frac{w_1}{W}\cdot F_1$.

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