BOOK FOUR

DEFINITIONS

1. A rectilineal figure is said to be *inscribed in a rectilineal figure* when the respective angles of the inscribed figure lie on the respective sides of that in which it is inscribed.

2. Similarly a figure is said to be *circumscribed about a figure* when the respective sides of the circumscribed figure pass through the respective angles of that about which it is circumscribed.

3. A rectilineal figure is said to be *inscribed in a circle* when each angle of the inscribed figure lies on the circumference of the circle.

4. A rectilineal figure is said to be *circumscribed about a circle*, when each side of the circumscribed figure touches the circumference of the circle.

5. Similarly a circle is said to be *inscribed in a figure* when the circumference of the circle touches each side of the figure in which it is inscribed.

6. A circle is said to be *circumscribed about a figure* when the circumference of the circle passes through each angle of the figure about which it is circumscribed.

7. A straight line is said to be *fitted into a circle* when its extremities are on the circumference of the circle.

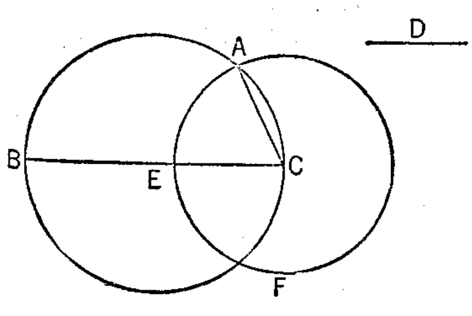
BOOK IV. PROPOSITIONS

Proposition 1

Into a given circle to fit a straight line equal to a given straight line which is not greater than the diameter of the circle.

Let ABC be the given circle, and D the given straight line not greater than the diameter of the circle;

thus it is required to fit into the circle ABC a straight line equal to the straight line D.



Let a diameter BC of the circle ABC be drawn.

Then, if BC is equal to D, that which was enjoined will have been done; for BC has been fitted into the circle ABC equal to the straight line D.

But, if BC is greater than D,

let CE be made equal to D, and with centre C and distance CE let the circle EAF be described;

let CA be joined.

Then, since the point C is the centre of the circle EAF, CA is equal to CE.

But CE is equal to D;

therefore D is also equal to CA.

Therefore into the given circle ABC there has been fitted CA equal to the given straight line D. Q. E. F.

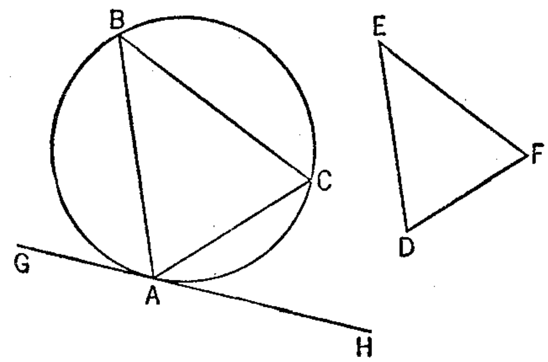
Proposition 2

In a given circle to inscribe a triangle equiangular with a given triangle.

Let ABC be the given circle, and DEF the given triangle; thus it is required to inscribe in the circle ABC a triangle equiangular with the triangle DEF.

Let GH be drawn touching the circle ABC at A [III. 16, Por.]; on the straight line AH, and at the point A on it, let the angle HAC be constructed equal to the angle DEF,

and on the straight line AG, and at the point A on it, let the angle GAB be constructed equal to the angle DFE; [I. 23]



let BC be joined.

Then, since a straight line AH touches the circle ABC, and from the point of contact at A the straight line AC is drawn across in the circle,

therefore the angle HAC is equal to the angle ABC in the alternate segment of the circle. [III. 32]

But the angle HAC is equal to the angle DEF;

therefore the angle ABC is also equal to the angle DEF.

For the same reason

the angle ACB is also equal to the angle DFE;

therefore the remaining angle BAC is also equal to the remaining angle EDF. [I. 32]

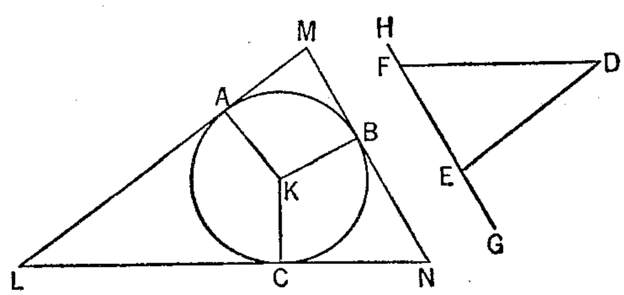
Therefore in the given circle there has been inscribed a triangle equiangular with the given triangle. Q. E. F.

Proposition 3

About a given circle to circumscribe a triangle equiangular with a given triangle.

Let ABC be the given circle, and DEF the given triangle; thus it is required to circumscribe about the circle ABC a triangle equiangular with the triangle DEF.

Let EF be produced in both directions to the points G, H, let the centre K of the circle ABC be taken [III. 1], and let the straight line KB be drawn across at random;



on the straight line KB, and at the point K on it, let the angle BKA be constructed equal to the angle DEG,

and the angle BKC equal to the angle DFH; [I. 23] and through the points A, B, C let LAM, MBN, NCL be drawn touching the circle ABC. [III. 16, Por.]

Now, since LM, MN, NL touch the circle ABC at the points A, B, C, and KA, KB, KC have been joined from the centre K to the points A, B, C,

therefore the angles at the points A, B, C are right. [III. 18]

And, since the four angles of the quadrilateral $AMBK$ are equal to four right angles, inasmuch as $AMBK$ is in fact divisible into two triangles,

and the angles KAM, KBM are right,

therefore the remaining angles AKB, AMB are equal to two right angles.

But the angles DEG, DEF are also equal to two right angles; [I. 13]

therefore the angles AKB, AMB are equal to the angles DEG, DEF,

of which the angle AKB is equal to the angle DEG;

therefore the angle AMB which remains is equal to the angle DEF which remains.

Similarly it can be proved that the angle LNB is also equal to the angle DFE;

therefore the remaining angle MLN is equal to the angle EDF. [I. 32]

Therefore the triangle LMN is equiangular with the triangle DEF; and it has been circumscribed about the circle ABC.

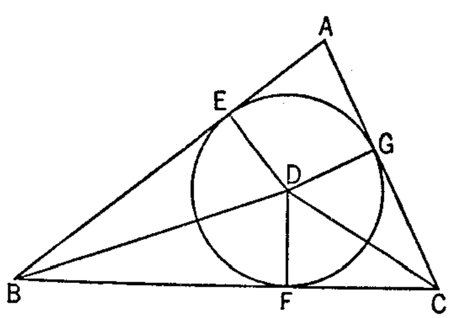
Therefore about a given circle there has been circumscribed a triangle equiangular with the given triangle. Q. E. F.

Proposition 4

In a given triangle to inscribe a circle.

Let ABC be the given triangle;

thus it is required to inscribe a circle in the triangle ABC.



Let the angles ABC, ACB be bisected by the straight lines BD, CD [I. 9], and let these meet one another at the point D; from D let DE, DF, DG be drawn perpendicular to the straight lines AB, BC, CA.

Now, since the angle ABD is equal to the angle CBD,

and the right angle BED is also equal to the right angle BFD,

EBD, FBD are two triangles having two angles equal to two angles and one side equal to one side, namely that subtending one of the equal angles, which is BD common to the triangles;

therefore they will also have the remaining sides equal to the remaining sides; [I. 26]

therefore DE is equal to DF.

For the same reason

DG is also equal to DF.

Therefore the three straight lines DE, DF, DG are equal to one another; therefore the circle described with centre D and distance one of the straight lines DE, DF, DG will pass also through the remaining points, and will touch the straight lines AB, BC, CA, because the angles at the points E, F, G are right.

For, if it cuts them, the straight line drawn at right angles to the diameter of

the circle from its extremity will be found to fall within the circle: which was proved absurd; [III. 16]
therefore the circle described with centre D and distance one of the straight lines DE, DF, DG will not cut the straight lines AB, BC, CA;
therefore it will touch them, and will be the circle inscribed in the triangle ABC. [IV. Def. 5]

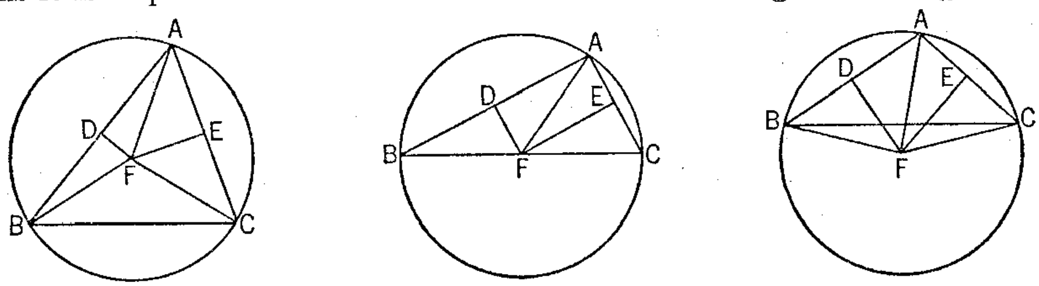
Let it be inscribed, as FGE.

Therefore in the given triangle ABC the circle EFG has been inscribed.

Proposition 5

About a given triangle to circumscribe a circle.

Let ABC be the given triangle;
thus it is required to circumscribe a circle about the given triangle ABC.



Let the straight lines AB, AC be bisected at the points D, E [I. 10], and from the points D, E let DF, EF be drawn at right angles to AB, AC;
they will then meet within the triangle ABC, or on the straight line BC, or outside BC.

First let them meet within at F, and let FB, FC, FA be joined.

Then, since AD is equal to DB,
and DF is common and at right angles,
therefore the base AF is equal to the base FB. [I. 4]

Similarly we can prove that
CF is also equal to AF;
so that FB is also equal to FC;
therefore the three straight lines FA, FB, FC are equal to one another.

Therefore the circle described with centre F and distance one of the straight lines FA, FB, FC will pass also through the remaining points, and the circle will have been circumscribed about the triangle ABC.

Let it be circumscribed, as ABC.

Next, let DF, EF meet on the straight line BC at F, as is the case in the second figure; and let AF be joined.

Then, similarly, we shall prove that the point F is the centre of the circle circumscribed about the triangle ABC.

Again, let DF, EF meet outside the triangle ABC at F, as is the case in the third figure, and let AF, BF, CF be joined.

Then again, since AD is equal to DB,
and DF is common and at right angles,
therefore the base AF is equal to the base BF. [I. 4]

Similarly we can prove that
CF is also equal to AF;
so that BF is also equal to FC;

therefore the circle described with centre F and distance one of the straight lines FA, FB, FC will pass also through the remaining points, and will have been circumscribed about the triangle ABC.

Therefore about the given triangle a circle has been circumscribed.

Q. E. F.

And it is manifest that, when the centre of the circle falls within the triangle, the angle BAC, being in a segment greater than the semicircle, is less than a right angle;

when the centre falls on the straight line BC, the angle BAC, being in a semicircle, is right;

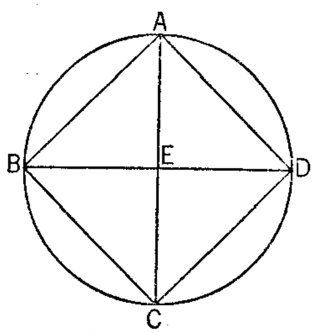
and when the centre of the circle falls outside the triangle, the angle BAC, being in a segment less than the semicircle, is greater than a right angle. [III. 31]

PROPOSITION 6

In a given circle to inscribe a square.

Let $ABCD$ be the given circle;

thus it is required to inscribe a square in the circle $ABCD$.



Let two diameters AC, BD of the circle $ABCD$ be drawn at right angles to one another, and let AB, BC, CD, DA be joined.

Then, since BE is equal to ED, for E is the centre, and EA is common and at right angles,

therefore the base AB is equal to the base AD. [I. 4]

For the same reason

each of the straight lines BC, CD is also equal to each of the straight lines AB, AD;

therefore the quadrilateral $ABCD$ is equilateral.

I say next that it is also right-angled.

For, since the straight line BD is a diameter of the circle $ABCD$,

therefore BAD is a semicircle;

therefore the angle BAD is right. [III. 31]

For the same reason

each of the angles ABC, BCD, CDA is also right;

therefore the quadrilateral $ABCD$ is right-angled.

But it was also proved equilateral;

therefore it is a square; [I. Def. 22]

and it has been inscribed in the circle $ABCD$.

Therefore in the given circle the square $ABCD$ has been inscribed. Q. E. F.

PROPOSITION 7

About a given circle to circumscribe a square.

Let $ABCD$ be the given circle;

thus it is required to circumscribe a square about the circle $ABCD$.

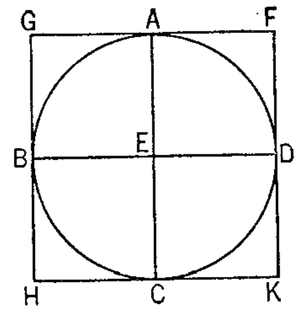
Let two diameters AC, BD of the circle $ABCD$ be drawn at right angles to one another, and through the points A, B, C, D let FG, GH, HK, KF be drawn touching the circle $ABCD$. [III. 16, Por.]

Then, since FG touches the circle $ABCD$,

and EA has been joined from the centre E to the point of contact at A,

therefore the angles at A are right. [III. 18]

For the same reason
the angles at the points B, C, D are also right.

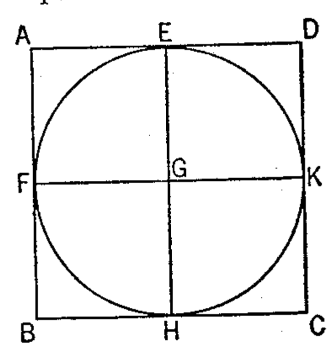


Now, since the angle AEB is right,
and the angle EBG is also right,
therefore GH is parallel to AC. [I. 28]
For the same reason
AC is also parallel to FK,
so that GH is also parallel to FK. [I. 30]
Similarly we can prove that
each of the straight lines GF, HK is parallel to BED.
Therefore GK, GC, AK, FB, BK are parallelograms;
therefore GF is equal to HK, and GH to FK. [I. 34]
And, since AC is equal to BD,
and AC is also equal to each of the straight lines GH, FK,
while BD is equal to each of the straight lines GF, HK, [I. 34]
therefore the quadrilateral $FGHK$ is equilateral.
I say next that it is also right-angled.
For, since $GBEA$ is a parallelogram,
and the angle AEB is right,
therefore the angle AGB is also right. [I. 34]
Similarly we can prove that
the angles at H, K, F are also right.
Therefore $FGHK$ is right-angled.
But it was also proved equilateral;
therefore it is a square;
and it has been circumscribed about the circle $ABCD$.
Therefore about the given circle a square has been circumscribed. Q. E. F.

Proposition 8

In a given square to inscribe a circle.

Let $ABCD$ be the given square;
thus it is required to inscribe a circle in the given square $ABCD$.



Let the straight lines AD, AB be bisected at the points E, F respectively, [I. 10]
through E let EH be drawn parallel to either AB or CD, and through F let FK be drawn parallel to either AD or BC; [I. 31]
therefore each of the figures AK, KB, AH, HD, AG, GC, BG, GD is a parallelogram, and their opposite sides are evidently equal. [I. 34]
Now, since AD is equal to AB,
and AE is half of AD, and AF half of AB,
therefore AE is equal to AF,
so that the opposite sides are also equal;
therefore FG is equal to GE.
Similarly we can prove that each of the straight lines GH, GK is equal to each of the straight lines FG, GE;
therefore the four straight lines GE, GF, GH, GK are equal to one another.

Therefore the circle described with centre G and distance one of the straight lines GE, GF, GH, GK will pass also through the remaining points.

And it will touch the straight lines AB, BC, CD, DA, because the angles at E, F, H, K are right.

For, if the circle cuts AB, BC, CD, DA, the straight line drawn at right angles to the diameter of the circle from its extremity will fall within the circle: which was proved absurd; [III. 16]
therefore the circle described with centre G and distance one of the straight lines GE, GF, GH, GK will not cut the straight lines AB, BC, CD, DA.

Therefore it will touch them, and will have been inscribed in the square $ABCD$.

Therefore in the given square a circle has been inscribed. Q. E. F.

Proposition 9

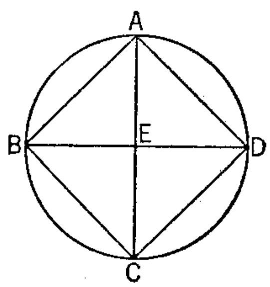
About a given square to circumscribe a circle.

Let $ABCD$ be the given square;

thus it is required to circumscribe a circle about the square $ABCD$.

For let AC, BD be joined, and let them cut one another at E.

Then, since DA is equal to AB, and AC is common,



therefore the two sides DA, AC are equal to the two sides BA, AC;

and the base DC is equal to the base BC;

therefore the angle DAC is equal to the angle BAC. [I. 8]

Therefore the angle DAB is bisected by AC.

Similarly we can prove that each of the angles ABC, BCD, CDA is bisected by the straight lines AC, DB.

Now, since the angle DAB is equal to the angle ABC,

and the angle EAB is half the angle DAB,

and the angle EBA half the angle ABC,

therefore the angle EAB is also equal to the angle EBA;

so that the side EA is also equal to EB. [I. 6]

Similarly we can prove that each of the straight lines EA, EB is equal to each of the straight lines EC, ED.

Therefore the four straight lines EA, EB, EC, ED are equal to one another.

Therefore the circle described with centre E and distance one of the straight lines EA, EB, EC, ED will pass also through the remaining points;

and it will have been circumscribed about the square $ABCD$.

Let it be circumscribed, as $ABCD$.

Therefore about the given square a circle has been circumscribed. Q. E. F.

Proposition 10

To construct an isosceles triangle having each of the angles at the base double of the remaining one.

Let any straight line AB be set out, and let it be cut at the point C so that the rectangle contained by AB, BC is equal to the square on CA; [II. 11]

with centre A and distance AB let the circle BDE be described,

and let there be fitted in the circle BDE the straight line BD equal to the straight line AC which is not greater than the diameter of the circle BDE. [IV. 1]

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