BOOK FIVE

DEFINITIONS

1. A magnitude is a *part* of a magnitude, the less of the greater, when it measures the greater.

2. The greater is a *multiple* of the less when it is measured by the less.

3. A *ratio* is a sort of relation in respect of size between two magnitudes of the same kind.

4. Magnitudes are said to *have a ratio* to one another which are capable, when multiplied, of exceeding one another.

5. Magnitudes are said to *be in the same ratio*, the first to the second and the third to the fourth, when, if any equimultiples whatever be taken of the first and third, and any equimultiples whatever of the second and fourth, the former equimultiples alike exceed, are alike equal to, or alike fall short of, the latter equimultiples respectively taken in corresponding order.

6. Let magnitudes which have the same ratio be called *proportional.*

7. When, of the equimultiples, the multiple of the first magnitude exceeds the multiple of the second, but the multiple of the third does not exceed the multiple of the fourth, then the first is said to *have a greater ratio* to the second than the third has to the fourth.

8. A proportion in three terms is the least possible.

9. When three magnitudes are proportional, the first is said to have to the third the *duplicate ratio* of that which it has to the second.

10. When four magnitudes are <continuously> proportional, the first is said to have to the fourth the *triplicate ratio* of that which it has to the second, and so on continually, whatever be the proportion.

11. The term *corresponding magnitudes* is used of antecedents in relation to antecedents, and of consequents in relation to consequents.

12. *Alternate ratio* means taking the antecedent in relation to the antecedent and the consequent in relation to the consequent.

13. *Inverse ratio* means taking the consequent as antecedent in relation to the antecedent as consequent.

14. *Composition of a ratio* means taking the antecedent together with the consequent as one in relation to the consequent by itself.

15. *Separation of a ratio* means taking the excess by which the antecedent exceeds the consequent in relation to the consequent by itself.

16. *Conversion of a ratio* means taking the antecedent in relation to the excess by which the antecedent exceeds the consequent.

17. A ratio *ex aequali* arises when, there being several magnitudes and another set equal to them in multitude which taken two and two are in the same proportion, as the first is to the last among the first magnitudes, so is the first to the last among the second magnitudes;

Or, in other words, it means taking the extreme terms by virtue of the removal of the intermediate terms.

18. A *perturbed proportion* arises when, there being three magnitudes and another set equal to them in multitude, as antecedent is to consequent among the first magnitudes, so is antecedent to consequent among the second magnitudes, while, as the consequent is to a third among the first magnitudes, so is a third to the antecedent among the second magnitudes.

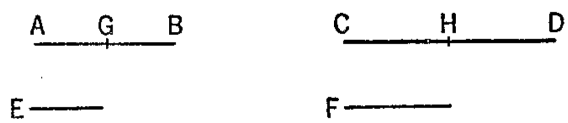
BOOK V. PROPOSITIONS

Proposition 1

If there be any number of magnitudes whatever which are, respectively, equimultiples of any magnitudes equal in multitude, then, whatever multiple one of the magnitudes is of one, that multiple also will all be of all.

Let any number of magnitudes whatever AB, CD be respectively equimultiples of any magnitudes E, F equal in multitude;

I say that, whatever multiple AB is of E, that multiple will AB, CD also be of E, F.



For, since AB is the same multiple of E that CD is of F, as many magnitudes as there are in AB equal to E, so many also are there in CD equal to F.

Let AB be divided into the magnitudes AG, GB equal to E,

and CD into CH, HD equal to F;

then the multitude of the magnitudes AG, GB will be equal to the multitude of the magnitudes CH, HD.

Now, since AG is equal to E, and CH to F,

therefore AG is equal to E, and AG, CH to E, F.

For the same reason

GB is equal to E, and GB, HD to E, F;

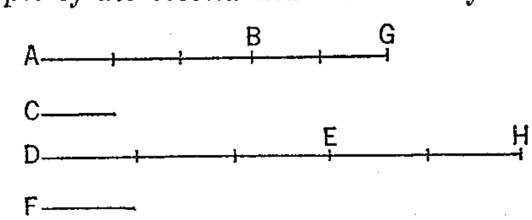
therefore, as many magnitudes as there are in AB equal to E, so many also are there in AB, CD equal to E, F;

therefore, whatever multiple AB is of E, that multiple will AB, CD also be of E, F.

Therefore etc. Q. E. D.

Proposition 2

If a first magnitude be the same multiple of a second that a third is of a fourth, and a fifth also be the same multiple of the second that a sixth is of the fourth, the sum of the first and fifth will also be the same multiple of the second that the sum of the third and sixth is of the fourth.



Let a first magnitude, AB, be the same multiple of a second, C, that a third, DE, is of a fourth, F, and let a fifth, BG, also be the same multiple of the second, C, that a sixth, EH, is of the fourth F;

I say that the sum of the first and fifth, AG, will be the same multiple of the second, C, that the sum of the third and sixth, DH, is of the fourth, F.

For, since AB is the same multiple of C that DE is of F, therefore, as many magnitudes as there are in AB equal to C, so many also are there in DE equal to F.

For the same reason also,

as many as there are in BG equal to C, so many are there also in EH equal to F; therefore, as many as there are in the whole AG equal to C, so many also are there in the whole DH equal to F.

Therefore, whatever multiple AG is of C, that multiple also is DH of F.

Therefore the sum of the first and fifth, AG, is the same multiple of the second, C, that the sum of the third and sixth, DH, is of the fourth, F.

Therefore etc. Q. E. D.

Proposition 3

If a first magnitude be the same multiple of a second that a third is of a fourth, and if equimultiples be taken of the first and third, then also ex aequali *the magnitudes taken will be equimultiples respectively, the one of the second, and the other of the fourth.*

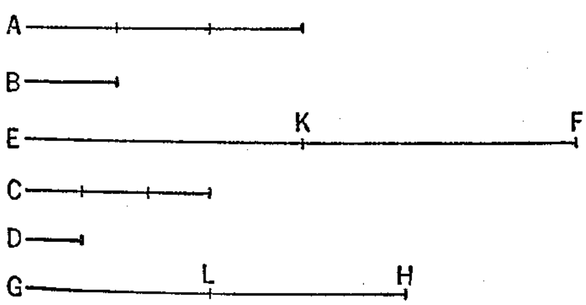
Let a first magnitude A be the same multiple of a second B that a third C is of a fourth D, and let equimultiples EF, GH be taken of A, C;

I say that EF is the same multiple of B that GH is of D.

For, since EF is the same multiple of A that GH is of C, therefore, as many magnitudes as there are in EF equal to A, so many also are there in GH equal to C.

Let EF be divided into the magnitudes EK, KF equal to A, and GH into the magnitudes GL, LH equal to C;

then the multitude of the magnitudes EK, KF will be equal to the multitude of the magnitudes GL, LH.



And, since A is the same multiple of B that C is of D, while EK is equal to A, and GL to C, therefore EK is the same multiple of B that GL is of D.

For the same reason

KF is the same multiple of B that LH is of D.

Since, then, a first magnitude EK is the same multiple of a second B that a third GL is of a fourth D,

and a fifth KF is also the same multiple of the second B that a sixth LH is of the fourth D,

therefore the sum of the first and fifth, EF, is also the same multiple of the second B that the sum of the third and sixth, GH, is of the fourth D. [v. 2]

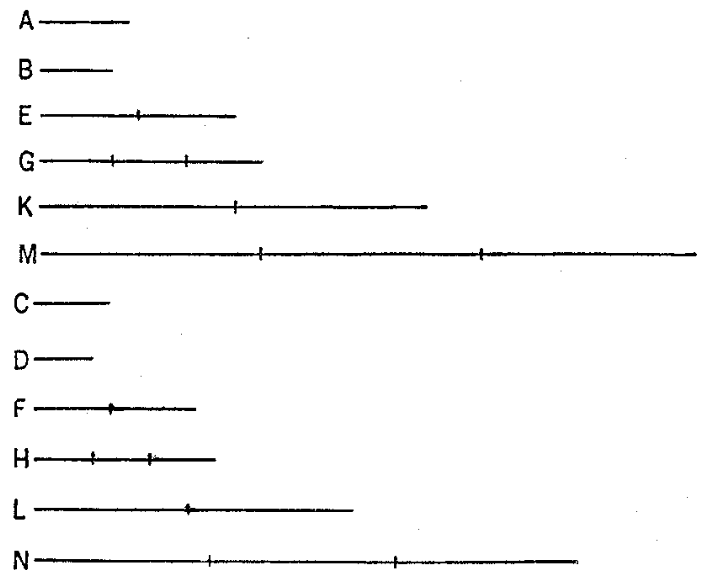
Therefore etc. Q. E. D.

Proposition 4

If a first magnitude have to a second the same ratio as a third to a fourth, any equimultiples whatever of the first and third will also have the same ratio to any equimultiples whatever of the second and fourth respectively, taken in corresponding order.

For let a first magnitude A have to a second B the same ratio as a third C to a fourth D; and let equimultiples E, F be taken of A, C, and G, H other, chance, equimultiples of B, D;

I say that, as E is to G, so is F to H.



For let equimultiples K, L be taken of E, F, and other, chance, equimultiples M, N of G, H.

Since E is the same multiple of A that F is of C, and equimultiples K, L of E, F have been taken, therefore K is the same multiple of A that L is of C. [v. 3]

For the same reason M is the same multiple of B that N is of D.

And, since, as A is to B, so is C to D,

and of A, C equimultiples K, L have been taken,
and of B, D other, chance, equimultiples M, N,
therefore, if K is in excess of M, L also is in excess of N,
if it is equal, equal, and if less, less. [v. Def. 5]
And K, L are equimultiples of E, F,
and M, N other, chance, equimultiples of G, H;
therefore, as E is to G, so is F to H. [v. Def. 5]

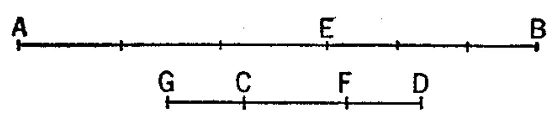
Therefore etc. Q. E. D.

Proposition 5

If a magnitude be the same multiple of a magnitude that a part subtracted is of a part subtracted, the remainder will also be the same multiple of the remainder that the whole is of the whole.

For let the magnitude AB be the same multiple of the magnitude CD that the part AE subtracted is of the part CF subtracted;

I say that the remainder EB is also the same multiple of the remainder FD that the whole AB is of the whole CD.



For, whatever multiple AE is of CF, let EB be made that multiple of CG.

Then, since AE is the same multiple of CF that EB is of GC,
therefore AE is the same multiple of CF that AB is of GF. [v. 1]

But, by the assumption, AE is the same multiple of CF that AB is of CD.

Therefore AB is the same multiple of each of the magnitudes GF, CD;
therefore GF is equal to CD.

Let CF be subtracted from each;

therefore the remainder GC is equal to the remainder FD.

And, since AE is the same multiple of CF that EB is of GC,

and GC is equal to DF,

therefore AE is the same multiple of CF that EB is of FD.

But, by hypothesis,

AE is the same multiple of CF that AB is of CD;

therefore EB is the same multiple of FD that AB is of CD.

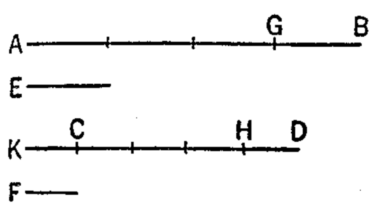
That is, the remainder EB will be the same multiple of the remainder FD that the whole AB is of the whole CD.

Therefore etc. Q. E. D.

Proposition 6

If two magnitudes be equimultiples of two magnitudes, and any magnitudes subtracted from them be equimultiples of the same, the remainders also are either equal to the same or equimultiples of them.

For let two magnitudes AB, CD be equimultiples of two magnitudes E, F, and let AG, CH subtracted from them be equimultiples of the same two E, F;



I say that the remainders also, GB, HD, are either equal to E, F or equimultiples of them.

For, first, let GB be equal to E;

I say that HD is also equal to F.

For let CK be made equal to F.

Since AG is the same multiple of E that CH is of F,

while GB is equal to E and KC to F,

therefore AB is the same multiple of E that KH is of F. [v. 2]

But, by hypothesis, AB is the same multiple of E that CD is of F;

therefore KH is the same multiple of F that CD is of F.

Since then each of the magnitudes KH, CD is the same multiple of F,

therefore KH is equal to CD.

Let CH be subtracted from each;

therefore the remainder KC is equal to the remainder HD.

But F is equal to KC;

therefore HD is also equal to F.

Hence, if GB is equal to E, HD is also equal to F.

Similarly we can prove that, even if GB be a multiple of E, HD is also the same multiple of F.

Therefore etc. Q. E. D.

Proposition 7

Equal magnitudes have to the same the same ratio, as also has the same to equal magnitudes.

Let A, B be equal magnitudes and C any other, chance, magnitude;

I say that each of the magnitudes A, B has the same ratio to C, and C has the same ratio to each of the magnitudes A, B.

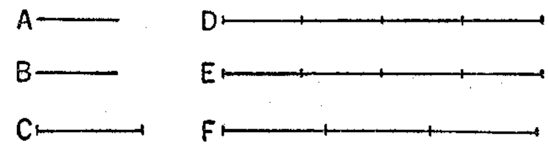
For let equimultiples D, E of A, B be taken, and of C another, chance, multiple F.

Then, since D is the same multiple of A that E is of B, while A is equal to B,

therefore D is equal to E.

But F is another, chance, magnitude.

If therefore D is in excess of F, E is also in excess of F, if equal to it, equal; and, if less, less.



And D, E are equimultiples of A, B, while F is another, chance, multiple of C; therefore, as A is to C, so is B to C. [v. Def. 5]

I say next that C also has the same ratio to each of the magnitudes A, B.

For, with the same construction, we can prove similarly that D is equal to E; and F is some other magnitude.

If therefore F is in excess of D, it is also in excess of E, if equal, equal; and, if less, less.

And F is a multiple of C, while D, E are other, chance, equimultiples of A, B; therefore, as C is to A, so is C to B. [v. Def. 5]

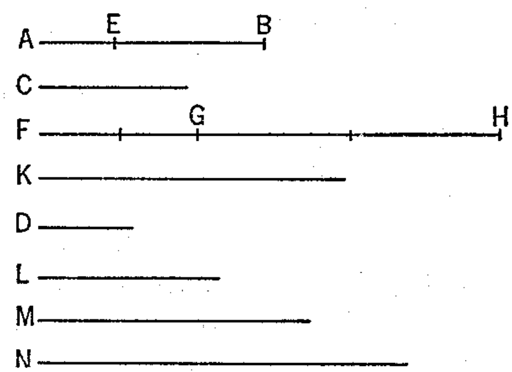
Therefore etc.

Porism. From this it is manifest that, if any magnitudes are proportional, they will also be proportional inversely. Q. E. D.

Proposition 8

Of unequal magnitudes, the greater has to the same a greater ratio than the less has; and the same has to the less a greater ratio than it has to the greater.

Let AB, C be unequal magnitudes, and let AB be greater; let D be another, chance, magnitude;



I say that AB has to D a greater ratio than C has to D, and D has to C a greater ratio than it has to AB.

For, since AB is greater than C, let BE be made equal to C;

then the less of the magnitudes AE, EB, if multiplied, will sometime be greater than D. [v. Def. 4]

First, let AE be less than EB; let AE be multiplied, and let FG be a multiple of it which is greater than D;

then, whatever multiple FG is of AE, let GH be made the same multiple of EB and K of C;

and let L be taken double of D, M triple of it, and successive multiples increasing by one, until what is taken is a multiple of D and the first that is greater than K. Let it be taken, and let it be N which is quadruple of D and the first multiple of it that is greater than K.

Then, since K is less than N first,

therefore K is not less than M.

And, since FG is the same multiple of AE that GH is of EB,

therefore FG is the same multiple of AE that FH is of AB. [v. 1]

But FG is the same multiple of AE that K is of C;

therefore FH is the same multiple of AB that K is of C;

therefore FH, K are equimultiples of AB, C.

Again, since GH is the same multiple of EB that K is of C,

and EB is equal to C,

therefore GH is equal to K.

But K is not less than M;

therefore neither is GH less than M.

And FG is greater than D;

therefore the whole FH is greater than D, M together.

But D, M together are equal to N, inasmuch as M is triple of D, and M, D together are quadruple of D, while N is also quadruple of D; whence M, D together are equal to N.

But FH is greater than M, D;

therefore FH is in excess of N,

while K is not in excess of N.

And FH, K are equimultiples of AB, C, while N is another, chance, multiple of D;

therefore AB has to D a greater ratio than C has to D. [v. Def. 7]

I say next, that D also has to C a greater ratio than D has to AB.

For, with the same construction, we can prove similarly that N is in excess of K, while N is not in excess of FH.

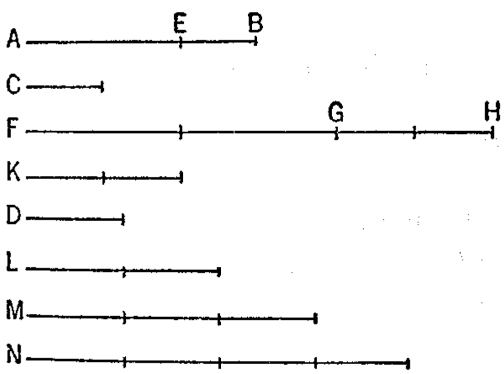
And N is a multiple of D,

while FH, K are other, chance, equimultiples of AB, C;

therefore D has to C a greater ratio than D has to AB. [v. Def. 7]

Again, let AE be greater than EB.

Then the less, EB, if multiplied, will sometime be greater than D. [v. Def. 4]



Let it be multiplied, and let GH be a multiple of EB and greater than D; and, whatever multiple GH is of EB, let FG be made the same multiple of AE, and K of C.

Then we can prove similarly that FH, K are equimultiples of AB, C; and, similarly, let N be taken a multiple of D but the first that is greater than FG, so that FG is again not less than M.

But GH is greater than D;

therefore the whole FH is in excess of D, M, that is, of N.

Now K is not in excess of N, inasmuch as FG also, which is greater than GH, that is, than K, is not in excess of N.

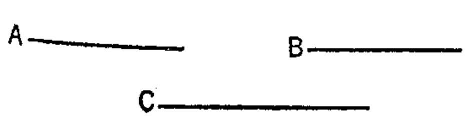
And in the same manner, by following the above argument, we complete the demonstration.

Therefore etc. Q. E. D.

Proposition 9

Magnitudes which have the same ratio to the same are equal to one another; and magnitudes to which the same has the same ratio are equal.

For let each of the magnitudes A, B have the same ratio to C;



I say that A is equal to B.

For, otherwise, each of the magnitudes A, B would not have had the same ratio to C; but it has; [v. 8]

therefore A is equal to B.

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