BOOK SIX

DEFINITIONS

1. *Similar rectilineal figures* are such as have their angles severally equal and the sides about the equal angles proportional.

2. A straight line is said to have been *cut in extreme and mean ratio* when, as the whole line is to the greater segment, so is the greater to the less.

3. The *height* of any figure is the perpendicular drawn from the vertex to the base.

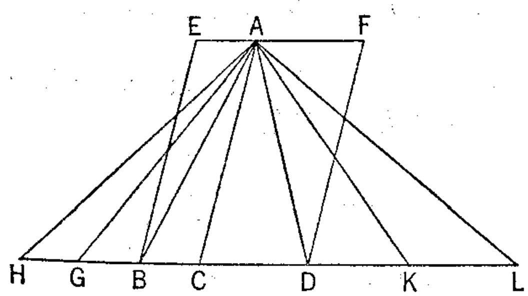
BOOK VI. PROPOSITIONS

Proposition 1

Triangles and parallelograms which are under the same height are to one another as their bases.

Let ABC, ACD be triangles and EC, CF parallelograms under the same height;

I say that, as the base BC is to the base CD, so is the triangle ABC to the triangle ACD, and the parallelogram EC to the parallelogram CF.



For let BD be produced in both directions to the points H, L and let [any number of straight lines] BG, GH be made equal to the base BC, and any number of straight lines DK, KL equal to the base CD;

let AG, AH, AK, AL be joined.

Then, since CB, BG, GH are equal to one another,

the triangles ABC, AGB, AHG are also equal to one another. [I. 38]

Therefore, whatever multiple the base HC is of the base BC, that multiple also is the triangle AHC of the triangle ABC.

For the same reason,

whatever multiple the base LC is of the base CD, that multiple also is the triangle ALC of the triangle ACD;

and, if the base HC is equal to the base CL, the triangle AHC is also equal to the triangle ACL, [I. 38]

if the base HC is in excess of the base CL, the triangle AHC is also in excess of the triangle ACL,

and, if less, less.

Thus, there being four magnitudes, two bases BC, CD and two triangles ABC, ACD,

equimultiples have been taken of the base BC and the triangle ABC, namely the base HC and the triangle AHC,
and of the base CD and the triangle ADC other, chance, equimultiples, namely the base LC and the triangle ALC;
and it has been proved that,
if the base HC is in excess of the base CL, the triangle AHC is also in excess of the triangle ALC;
if equal, equal; and, if less, less.

Therefore, as the base BC is to the base CD, so is the triangle ABC to the triangle ACD. [v. Def. 5]

Next, since the parallelogram EC is double of the triangle ABC, [I. 41]
and the parallelogram FC is double of the triangle ACD,
while parts have the same ratio as the same multiples of them, [v. 15]
therefore, as the triangle ABC is to the triangle ACD, so is the parallelogram EC to the parallelogram FC.

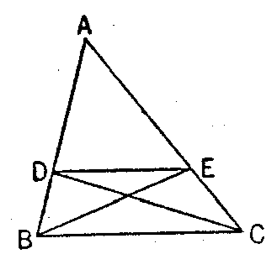
Since, then, it was proved that, as the base BC is to CD, so is the triangle ABC to the triangle ACD,
and, as the triangle ABC is to the triangle ACD, so is the parallelogram EC to the parallelogram CF,
therefore also, as the base BC is to the base CD, so is the parallelogram EC to the parallelogram FC. [v. 11]

Therefore etc. Q. E. D.

Proposition 2

If a straight line be drawn parallel to one of the sides of a triangle, it will cut the sides of the triangle proportionally; and, if the sides of the triangle be cut proportionally, the line joining the points of section will be parallel to the remaining side of the triangle.

For let DE be drawn parallel to BC, one of the sides of the triangle ABC;
I say that, as BD is to DA, so is CE to EA.



For let BE, CD be joined.

Therefore the triangle BDE is equal to the triangle CDE;
for they are on the same base DE and in the same parallels DE, BC. [I. 38]

And the triangle ADE is another area.

But equals have the same ratio to the same; [v. 7]
therefore, as the triangle BDE is to the triangle ADE, so is the triangle CDE to the triangle ADE.

But, as the triangle BDE is to ADE, so is BD to DA;
for, being under the same height, the perpendicular drawn from E to AB, they are to one another as their bases. [VI. 1]

For the same reason also,
as the triangle CDE is to ADE, so is CE to EA.

Therefore also, as BD is to DA, so is CE to EA. [v. 11]

Again, let the sides AB, AC of the triangle ABC be cut proportionally, so that, as BD is to DA, so is CE to EA; and let DE be joined.

I say that DE is parallel to BC.

For, with the same construction,

since, as BD is to DA, so is CE to EA,

but, as BD is to DA, so is the triangle BDE to the triangle ADE,

and, as CE is to EA, so is the triangle CDE to the triangle ADE, [VI. 1]

therefore also,

as the triangle BDE is to the triangle ADE, so is the triangle CDE to the triangle ADE. [V. 11]

Therefore each of the triangles BDE, CDE has the same ratio to ADE.

Therefore the triangle BDE is equal to the triangle CDE; [V. 9]

and they are on the same base DE.

But equal triangles which are on the same base are also in the same parallels. [I. 39]

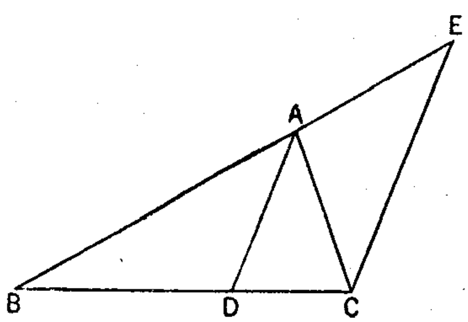
Therefore DE is parallel to BC.

Therefore etc. Q. E. D.

Proposition 3

If an angle of a triangle be bisected and the straight line cutting the angle cut the base also, the segments of the base will have the same ratio as the remaining sides of the triangle; and, if the segments of the base have the same ratio as the remaining sides of the triangle, the straight line joined from the vertex to the point of section will bisect the angle of the triangle.

Let ABC be a triangle, and let the angle BAC be bisected by the straight line AD;



I say that, as BD is to CD, so is BA to AC.

For let CE be drawn through C parallel to DA, and let BA be carried through and meet it at E.

Then, since the straight line AC falls upon the parallels AD, EC,

the angle ACE is equal to the angle CAD. [I. 29]

But the angle CAD is by hypothesis equal to the angle BAD;

therefore the angle BAD is also equal to the angle ACE.

Again, since the straight line BAE falls upon the parallels AD, EC,

the exterior angle BAD is equal to the interior angle AEC. [I. 29]

But the angle ACE was also proved equal to the angle BAD;

therefore the angle ACE is also equal to the angle AEC,

so that the side AE is also equal to the side AC. [I. 6]

And, since AD has been drawn parallel to EC, one of the sides of the triangle BCE,

therefore, proportionally, as BD is to DC, so is BA to AE.

But AE is equal to AC; [VI. 2]

therefore, as BD is to DC, so is BA to AC.

Again, let BA be to AC as BD to DC, and let AD be joined;

I say that the angle BAC has been bisected by the straight line AD.

For, with the same construction,

since, as BD is to DC, so is BA to AC,

and also, as BD is to DC, so is BA to AE: for AD has been drawn parallel to

EC, one of the sides of the triangle BCE: [VI. 2]

therefore also, as BA is to AC, so is BA to AE. [V. 11]

Therefore AC is equal to AE, [V. 9]

so that the angle AEC is also equal to the angle ACE. [I. 5]

But the angle AEC is equal to the exterior angle BAD, [I. 29]

and the angle ACE is equal to the alternate angle CAD; [*id.*]

therefore the angle BAD is also equal to the angle CAD.

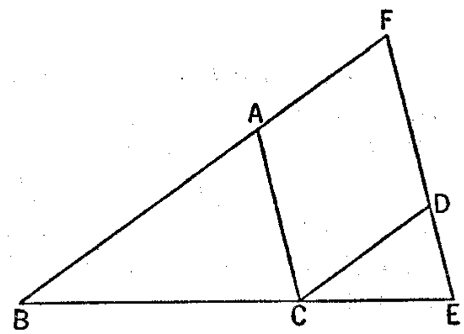
Therefore the angle BAC has been bisected by the straight line AD.

Therefore etc. Q. E. D.

Proposition 4

In equiangular triangles the sides about the equal angles are proportional, and those are corresponding sides which subtend the equal angles.

Let ABC, DCE be equiangular triangles having the angle ABC equal to the angle DCE, the angle BAC to the angle CDE, and further the angle ACB to the angle CED;



I say that in the triangles ABC, DCE the sides about the equal angles are proportional, and those are corresponding sides which subtend the equal angles.

For let BC be placed in a straight line with CE.

Then, since the angles ABC, ACB are less than two right angles, [I. 17]

and the angle ACB is equal to the angle DEC,

therefore the angles ABC, DEC are less than two right angles;

therefore BA, ED, when produced, will meet. [I. Post. 5]

Let them be produced and meet at F.

Now, since the angle DCE is equal to the angle ABC,

BF is parallel to CD. [I. 28]

Again, since the angle ACB is equal to the angle DEC,

AC is parallel to FE. [I. 28]

Therefore $FACD$ is a parallelogram;

therefore FA is equal to DC, and AC to FD. [I. 34]

And, since AC has been drawn parallel to FE, one side of the triangle FBE,

therefore, as BA is to AF, so is BC to CE. [VI. 2]

But AF is equal to CD;

therefore, as BA is to CD, so is BC to CE,

and alternately, as AB is to BC, so is DC to CE. [V. 16]

Again, since CD is parallel to BF,

therefore, as BC is to CE, so is FD to DE. [VI. 2]

But FD is equal to AC;

therefore, as BC is to CE, so is AC to DE,

and alternately, as BC is to CA, so is CE to ED. [V. 16]

Since, then, it was proved that,

as AB is to BC, so is DC to CE,

and, as BC is to CA, so is CE to ED;

therefore, *ex aequali*, as BA is to AC, so is CD to DE. [V. 22]

Therefore etc. Q. E. D.

Proposition 5

If two triangles have their sides proportional, the triangles will be equiangular and will have those angles equal which the corresponding sides subtend.

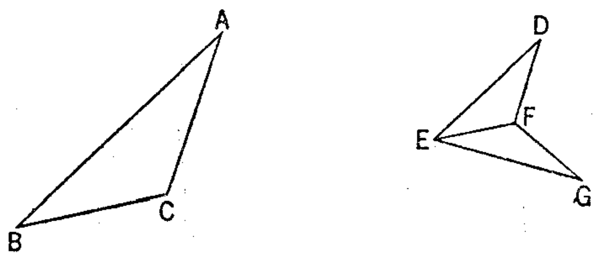
Let ABC, DEF be two triangles having their sides proportional, so that,

as AB is to BC, so is DE to EF,
as BC is to CA, so is EF to FD,
and further, as BA is to AC, so is ED to DF;

I say that the triangle ABC is equiangular with the triangle DEF, and they will have those angles equal which the corresponding sides subtend, namely the angle ABC to the angle DEF, the angle BCA to the angle EFD, and further the angle BAC to the angle EDF.

For on the straight line EF, and at the points E, F on it, let there be constructed the angle FEG equal to the angle ABC, and the angle EFG equal to the angle ACB; [I. 23]
therefore the remaining angle at A is equal to the remaining angle at G. [I. 32]

Therefore the triangle ABC is equiangular with the triangle GEF.



Therefore in the triangles ABC, GEF the sides about the equal angles are proportional, and those are corresponding sides which subtend the equal angles; [VI. 4]
therefore, as AB is to BC, so is GE to EF.

But, as AB is to BC, so by hypothesis is DE to EF;

therefore, as DE is to EF, so is GE to EF. [V. 11]

Therefore each of the straight lines DE, GE has the same ratio to EF;

therefore DE is equal to GE. [V. 9]

For the same reason

DF is also equal to GF.

Since then DE is equal to EG,

and EF is common,
the two sides DE, EF are equal to the two sides GE, EF;
and the base DF is equal to the base FG;
therefore the angle DEF is equal to the angle GEF, [I. 8]
and the triangle DEF is equal to the triangle GEF,

and the remaining angles are equal to the remaining angles, namely those which the equal sides subtend. [I. 4]

Therefore the angle DFE is also equal to the angle GFE,

and the angle EDF to the angle EGF.

And, since the angle FED is equal to the angle GEF,

while the angle GEF is equal to the angle ABC,
therefore the angle ABC is also equal to the angle DEF.

For the same reason

the angle ACB is also equal to the angle DFE,
and further, the angle at A to the angle at D;
therefore the triangle ABC is equiangular with the triangle DEF.

Therefore etc. Q. E. D.

Proposition 6

If two triangles have one angle equal to one angle and the sides about the equal angles proportional, the triangles will be equiangular and will have those angles equal which the corresponding sides subtend.

Let ABC, DEF be two triangles having one angle BAC equal to one angle EDF and the sides about the equal angles proportional, so that,

as BA is to AC, so is ED to DF;

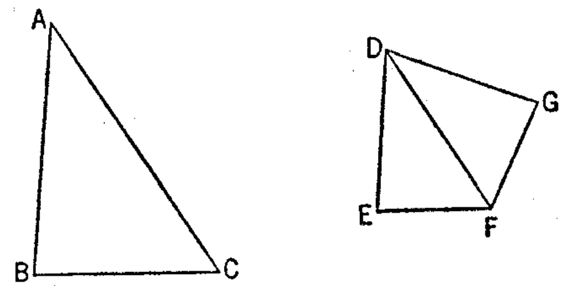
I say that the triangle ABC is equiangular with the triangle DEF, and will have the angle ABC equal to the angle DEF, and the angle ACB to the angle DFE.

For on the straight line DF, and at the points D, F on it, let there be constructed the angle FDG equal to either of the angles BAC, EDF, and the angle DFG equal to the angle ACB; [I. 23]
therefore the remaining angle at B is equal to the remaining angle at G. [I. 32]

Therefore the triangle ABC is equiangular with the triangle DGF.

Therefore, proportionally, as BA is to AC, so is GD to DF. [VI. 4]

But, by hypothesis, as BA is to AC, so also is ED to DF;
therefore also, as ED is to DF, so is GD to DF. [V. 11]



Therefore ED is equal to DG; [V. 9]

and DF is common;

therefore the two sides ED, DF are equal to the two sides GD, DF; and the angle EDF is equal to the angle GDF;

therefore the base EF is equal to the base GF,

and the triangle DEF is equal to the triangle DGF,

and the remaining angles will be equal to the remaining angles, namely those which the equal sides subtend. [I. 4]

Therefore the angle DFG is equal to the angle DFE,

and the angle DGF to the angle DEF.

But the angle DFG is equal to the angle ACB;

therefore the angle ACB is also equal to the angle DFE.

And, by hypothesis, the angle BAC is also equal to the angle EDF;
therefore the remaining angle at B is also equal to the remaining angle at E; [I. 32]

therefore the triangle ABC is equiangular with the triangle DEF.

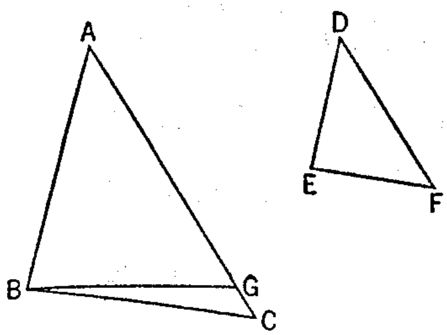
Therefore etc. Q. E. D.

Proposition 7

If two triangles have one angle equal to one angle, the sides about other angles proportional, and the remaining angles either both less or both not less than a right angle, the triangles will be equiangular and will have those angles equal, the sides about which are proportional.

Let ABC, DEF be two triangles having one angle equal to one angle, the angle BAC to the angle EDF, the sides about other angles ABC, DEF proportional, so that, as AB is to BC, so is DE to EF, and, first, each of the remaining angles at C, F less than a right angle;

I say that the triangle ABC is equiangular with the triangle DEF, the angle ABC will be equal to the angle DEF, and the remaining angle, namely the angle at C, equal to the remaining angle, the angle at F.



For, if the angle ABC is unequal to the angle DEF, one of them is greater.

Let the angle ABC be greater;
and on the straight line AB, and at the point B on it, let the angle ABG be constructed equal to the angle DEF. [I. 23]

Then, since the angle A is equal to D,
and the angle ABG to the angle DEF,
therefore the remaining angle AGB is equal to the remaining angle DFE. [I. 32]

Therefore the triangle ABG is equiangular with the triangle DEF.

Therefore, as AB is to BG, so is DE to EF. [VI. 4]

But, as DE is to EF, so by hypothesis is AB to BC;
therefore AB has the same ratio to each of the straight lines BC, BG; [V. 11]
therefore BC is equal to BG, [V. 9]
so that the angle at C is also equal to the angle BGC. [I. 5]

But, by hypothesis, the angle at C is less than a right angle;
therefore the angle BGC is also less than a right angle;
so that the angle AGB adjacent to it is greater than a right angle. [I. 13]

And it was proved equal to the angle at F;
therefore the angle at F is also greater than a right angle.

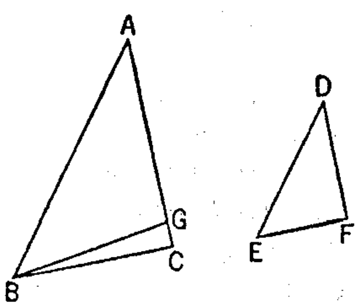
But it is by hypothesis less than a right angle: which is absurd.

Therefore the angle ABC is not unequal to the angle DEF;
therefore it is equal to it.

But the angle at A is also equal to the angle at D;
therefore the remaining angle at C is equal to the remaining angle at F. [I. 32]

Therefore the triangle ABC is equiangular with the triangle DEF.

But, again, let each of the angles at C, F be supposed not less than a right angle;



I say again that, in this case too, the triangle ABC is equiangular with the triangle DEF.

For, with the same construction, we can prove similarly that

BC is equal to BG;

so that the angle at C is also equal to the angle BGC. [I. 5]

But the angle at C is not less than a right angle;
therefore neither is the angle BGC less than a right angle.

Thus in the triangle BGC the two angles are not less than two right angles: which is impossible. [I. 17]

Therefore, once more, the angle ABC is not unequal to the angle DEF;
therefore it is equal to it.

But the angle at A is also equal to the angle at D;
therefore the remaining angle at C is equal to the remaining angle at F. [I. 32]

Therefore the triangle ABC is equiangular with the triangle DEF.

Therefore etc.

Q. E. D.

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