BOOK ELEVEN

DEFINITIONS

1. A *solid* is that which has length, breadth, and depth.
2. An extremity of a solid is a surface.
3. A *straight line* is *at right angles to a plane,* when it makes right angles with all the straight lines which meet it and are in the plane.
4. A *plane* is *at right angles to a plane* when the straight lines drawn, in one of the planes, at right angles to the common section of the planes are at right angles to the remaining plane.
5. The *inclination of a straight line to a plane* is, assuming a perpendicular drawn from the extremity of the straight line which is elevated above the plane to the plane, and a straight line joined from the point thus arising to the extremity of the straight line which is in the plane, the angle contained by the straight line so drawn and the straight line standing up.
6. The *inclination of a plane to a plane* is the acute angle contained by the straight lines drawn at right angles to the common section at the same point, one in each of the planes.
7. A plane is said to be *similarly inclined* to a plane as another is to another when the said angles of the inclinations are equal to one another.
8. *Parallel planes* are those which do not meet.
9. *Similar solid figures* are those contained by similar planes equal in multitude.
10. *Equal and similar solid figures* are those contained by similar planes equal in multitude and in magnitude.
11. A *solid angle* is the inclination constituted by more than two lines which meet one another and are not in the same surface, towards all the lines.

Otherwise: A *solid angle* is that which is contained by more than two plane angles which are not in the same plane and are constructed to one point.

12. A *pyramid* is a solid figure, contained by planes, which is constructed from one plane to one point.
13. A *prism* is a solid figure contained by planes two of which, namely those which are opposite, are equal, similar and parallel, while the rest are parallelograms.
14. When, the diameter of a semicircle remaining fixed, the semicircle is carried round and restored again to the same position from which it began to be moved, the figure so comprehended is a *sphere.*
15. The *axis of the sphere* is the straight line which remains fixed and about which the semicircle is turned.
16. The *centre of the sphere* is the same as that of the semicircle.
17. A *diameter of the sphere* is any straight line drawn through the centre and

terminated in both directions by the surface of the sphere.

18. When, one side of those about the right angle in a right-angled triangle remaining fixed, the triangle is carried round and restored again to the same position from which it began to be moved, the figure so comprehended is a *cone*.

And, if the straight line which remains fixed be equal to the remaining side about the right angle which is carried round, the cone will be *right-angled*; if less, *obtuse-angled*; and if greater, *acute-angled*.

19. The *axis of the cone* is the straight line which remains fixed and about which the triangle is turned.

20. And the *base* is the circle described by the straight line which is carried round.

21. When, one side of those about the right angle in a rectangular parallelogram remaining fixed, the parallelogram is carried round and restored again to the same position from which it began to be moved, the figure so comprehended is a *cylinder*.

22. The *axis of the cylinder* is the straight line which remains fixed and about which the parallelogram is turned.

23. And the *bases* are the circles described by the two sides opposite to one another which are carried round.

24. *Similar cones and cylinders* are those in which the axes and the diameters of the bases are proportional.

25. A *cube* is a solid figure contained by six equal squares.

26. An *octahedron* is a solid figure contained by eight equal and equilateral triangles.

27. An *icosahedron* is a solid figure contained by twenty equal and equilateral triangles.

28. A *dodecahedron* is a solid figure contained by twelve equal, equilateral, and equiangular pentagons.

BOOK XI. PROPOSITIONS

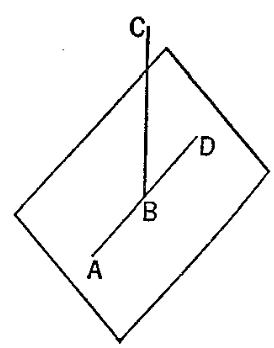
Proposition 1

A part of a straight line cannot be in the plane of reference and a part in a plane more elevated.

For, if possible, let a part AB of the straight line ABC be in the plane of reference, and a part BC in a plane more elevated.

There will then be in the plane of reference some straight line continuous with AB in a straight line.

Let it be BD;
therefore AB is a common segment of the two straight lines ABC, ABD:
which is impossible, inasmuch as, if we describe a circle with centre B and distance AB, the diameters will cut off unequal circumferences of the circle.



Therefore a part of a straight line cannot be in the plane of reference, and a part in a plane more elevated.

Q. E. D.

PROPOSITION 2

If two straight lines cut one another, they are in one plane, and every triangle is in one plane.

For let the two straight lines AB, CD cut one another at the point E;

I say that AB, CD are in one plane, and every triangle is in one plane.

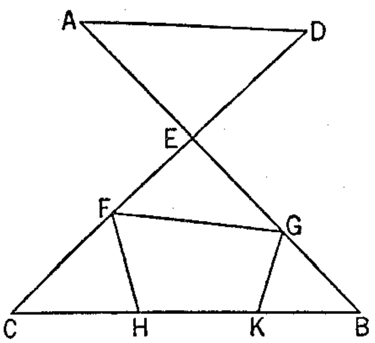
For let points F, G be taken at random on EC, EB,

let CB, FG be joined,

and let FH, GK be drawn across;

I say first that the triangle ECB is in one plane.

For, if part of the triangle ECB, either FHC or GBK, is in the plane of reference, and the rest in another,



a part also of one of the straight lines EC, EB will be in the plane of reference, and a part in another.

But, if the part $FCBG$ of the triangle ECB be in the plane of reference, and the rest in another, a part also of both the straight lines EC, EB will be in the plane of reference and a part in another:

which was proved absurd. [XI. 1]

Therefore the triangle ECB is in one plane.

But, in whatever plane the triangle ECB is, in that plane also is each of the straight lines EC, EB, and, in whatever plane each of the straight lines EC, EB is, in that plane are AB, CD also. [XI. 1]

Therefore the straight lines AB, CD are in one plane,

and every triangle is in one plane. Q. E. D.

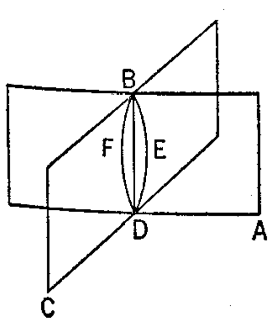
PROPOSITION 3

If two planes cut one another, their common section is a straight line.

For let the two planes AB, BC cut one another,

and let the line DB be their common section;

I say that the line DB is a straight line.



For, if not, from D to B let the straight line DEB be joined in the plane AB,

and in the plane BC the straight line DFB.

Then the two straight lines DEB, DFB will have the same extremities, and will clearly enclose an area:

which is absurd.

Therefore DEB, DFB are not straight lines.

Similarly we can prove that neither will there be any other straight line joined from D to B except DB the common section of the planes AB, BC.

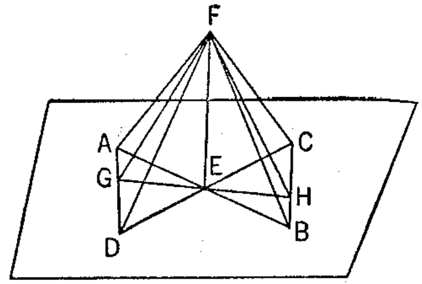
Therefore etc. Q. E. D.

PROPOSITION 4

If a straight line be set up at right angles to two straight lines which cut one another, at their common point of section, it will also be at right angles to the plane through them.

For let a straight line EF be set up at right angles to the two straight lines AB, CD, which cut one another at the point E, from E;

I say that EF is also at right angles to the plane through AB, CD.



For let AE, EB, CE, ED be cut off equal to one another,

and let any straight line GEH be drawn across through E, at random;

let AD, CB be joined,

and further, let FA, FG, FD, FC, FH, FB be joined from the point F taken at random <on EF>.

Now, since the two straight lines AE, ED are equal to the two straight lines CE, EB, and contain equal angles, [I. 15]

therefore the base AD is equal to the base CB,

and the triangle AED will be equal to the triangle CEB; [I. 4]

so that the angle DAE is also equal to the angle EBC.

But the angle AEG is also equal to the angle BEH; [I. 15]

therefore AGE, BEH are two triangles which have two angles equal to two angles respectively, and one side equal to one side, namely that adjacent to the equal angles, that is to say, AE to EB;

therefore they will also have the remaining sides equal to the remaining sides. [I. 26]

Therefore GE is equal to EH, and AG to BH.

And, since AE is equal to EB,

while FE is common and at right angles,

therefore the base FA is equal to the base FB. [I. 4]

For the same reason

FC is also equal to FD.

And, since AD is equal to CB,

and FA is also equal to FB,

the two sides FA, AD are equal to the two sides FB, BC respectively;

and the base FD was proved equal to the base FC;

therefore the angle FAD is also equal to the angle FBC. [I. 8]

And since, again, AG was proved equal to BH,

and further, FA also equal to FB,

the two sides FA, AG are equal to the two sides FB, BH.

And the angle FAG was proved equal to the angle FBH;

therefore the base FG is equal to the base FH. [I. 4]

Now since, again, GE was proved equal to EH,

and EF is common,

the two sides GE, EF are equal to the two sides HE, EF;

and the base FG is equal to the base FH;

therefore the angle GEF is equal to the angle HEF. [I. 8]

Therefore each of the angles GEF, HEF is right.

Therefore FE is at right angles to GH drawn at random through E.

Similarly we can prove that FE will also make right angles with all the straight lines which meet it and are in the plane of reference.

But a straight line is at right angles to a plane when it makes right angles with all the straight lines which meet it and are in that same plane; [XI. Def. 3]

therefore *FE* is at right angles to the plane of reference.

But the plane of reference is the plane through the straight lines *AB*, *CD*.

Therefore *FE* is at right angles to the plane through *AB*, *CD*.

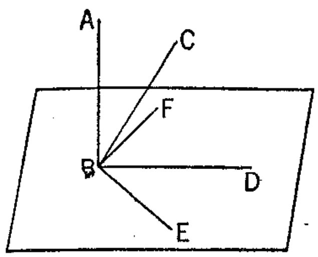
Therefore etc. Q. E. D.

Proposition 5

If a straight line be set up at right angles to three straight lines which meet one another, at their common point of section, the three straight lines are in one plane.

For let a straight line *AB* be set up at right angles to the three straight lines *BC*, *BD*, *BE*, at their point of meeting at *B*;

I say that *BC*, *BD*, *BE* are in one plane.



For suppose they are not, but, if possible, let *BD*, *BE* be in the plane of reference and *BC* in one more elevated;

let the plane through *AB*, *BC* be produced;

it will thus make, as common section in the plane of reference, a straight line. [XI. 3]

Let it make *BF*.

Therefore the three straight lines *AB*, *BC*, *BF* are in one plane, namely that drawn through *AB*, *BC*.

Now, since *AB* is at right angles to each of the straight lines *BD*, *BE*, therefore *AB* is also at right angles to the plane through *BD*, *BE*. [XI. 4]

But the plane through *BD*, *BE* is the plane of reference;

therefore *AB* is at right angles to the plane of reference.

Thus *AB* will also make right angles with all the straight lines which meet it and are in the plane of reference. [XI. Def. 3]

But *BF* which is in the plane of reference meets it;

therefore the angle *ABF* is right.

But, by hypothesis, the angle *ABC* is also right;

therefore the angle *ABF* is equal to the angle *ABC*.

And they are in one plane:

which is impossible.

Therefore the straight line *BC* is not in a more elevated plane;

therefore the three straight lines *BC*, *BD*, *BE* are in one plane.

Therefore, if a straight line be set up at right angles to three straight lines, at their point of meeting, the three straight lines are in one plane. Q. E. D.

Proposition 6

If two straight lines be at right angles to the same plane, the straight lines will be parallel.

For let the two straight lines *AB*, *CD* be at right angles to the plane of reference;

I say that *AB* is parallel to *CD*.

For let them meet the plane of reference at the points *B*, *D*,

let the straight line *BD* be joined,

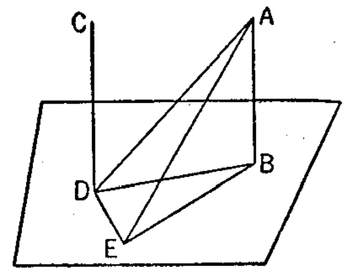
let *DE* be drawn, in the plane of reference, at right angles to *BD*,

let *DE* be made equal to *AB*,

and let BE, AE, AD be joined.

Now, since AB is at right angles to the plane of reference, it will also make right angles with all the straight lines which meet it and are in the plane of reference. [XI. Def. 3]

But each of the straight lines BD, BE is in the plane of reference and meets AB;



therefore each of the angles ABD, ABE is right.

For the same reason

each of the angles CDB, CDE is also right.

And, since AB is equal to DE,

and BD is common,

the two sides AB, BD are equal to the two sides ED, DB;

and they include right angles;

therefore the base AD is equal to the base BE. [I. 4]

And, since AB is equal to DE,

while AD is also equal to BE,

the two sides AB, BE are equal to the two sides ED, DA;

and AE is their common base;

therefore the angle ABE is equal to the angle EDA. [I. 8]

But the angle ABE is right;

therefore the angle EDA is also right;

therefore ED is at right angles to DA.

But it is also at right angles to each of the straight lines BD, DC; therefore ED is set up at right angles to the three straight lines BD, DA, DC at their point of meeting;

therefore the three straight lines BD, DA, DC are in one plane. [XI. 5]

But, in whatever plane DB, DA are, in that plane is AB also,

for every triangle is in one plane; [XI. 2]

therefore the straight lines AB, BD, DC are in one plane.

And each of the angles ABD, BDC is right;

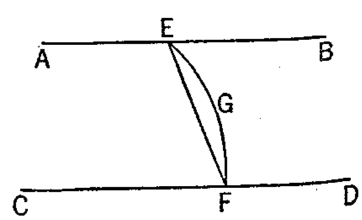
therefore AB is parallel to CD. [I. 28]

Therefore etc. Q. E. D.

Proposition 7

If two straight lines be parallel and points be taken at random on each of them, the straight line joining the points is in the same plane with the parallel straight lines.

Let AB, CD be two parallel straight lines, and let points E, F be taken at random on them respectively;



I say that the straight line joining the points E, F is in the same plane with the parallel straight lines.

For suppose it is not, but, if possible, let it be in a more elevated plane as EGF,

and let a plane be drawn through EGF;

it will then make, as section in the plane of reference, a straight line. [XI. 3]

Let it make it, as EF;

therefore the two straight lines EGF, EF will enclose an area:

which is impossible.

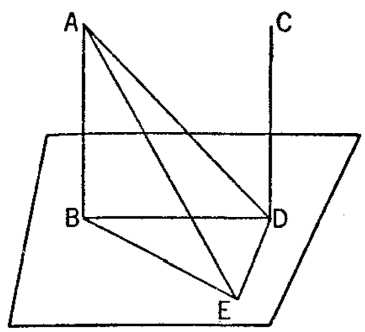
Therefore the straight line joined from E to F is not in a plane more elevated; therefore the straight line joined from E to F is in the plane through the parallel straight lines AB, CD.

Therefore etc. Q. E. D.

Proposition 8

If two straight lines be parallel, and one of them be at right angles to any plane, the remaining one will also be at right angles to the same plane.

Let AB, CD be two parallel straight lines,



and let one of them, AB, be at right angles to the plane of reference;

I say that the remaining one, CD, will also be at right angles to the same plane.

For let AB, CD meet the plane of reference at the points B, D,

and let BD be joined;

therefore AB, CD, BD are in one plane. [XI. 7]

Let DE be drawn, in the plane of reference, at right angles to BD,

let DE be made equal to AB,

and let BE, AE, AD be joined.

Now, since AB is at right angles to the plane of reference, therefore AB is also at right angles to all the straight lines which meet it and are in the plane of reference; [XI. Def. 3]

therefore each of the angles ABD, ABE is right.

And, since the straight line BD has fallen on the parallels AB, CD,

therefore the angles ABD, CDB are equal to two right angles. [I. 29]

But the angle ABD is right;

therefore the angle CDB is also right;

therefore CD is at right angles to BD.

And, since AB is equal to DE,

and BD is common,

the two sides AB, BD are equal to the two sides ED, DB;

and the angle ABD is equal to the angle EDB,

for each is right;

therefore the base AD is equal to the base BE.

And, since AB is equal to DE,

and BE to AD,

the two sides AB, BE are equal to the two sides ED, DA respectively,

and AE is their common base;

therefore the angle ABE is equal to the angle EDA.

But the angle ABE is right;

therefore the angle EDA is also right;

therefore ED is at right angles to AD.

But it is also at right angles to DB;

therefore ED is also at right angles to the plane through BD, DA. [XI. 4]

Therefore ED will also make right angles with all the straight lines which meet it and are in the plane through BD, DA.

But DC is in the plane through BD, DA, inasmuch as AB, BD are in the

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