BOOK THIRTEEN

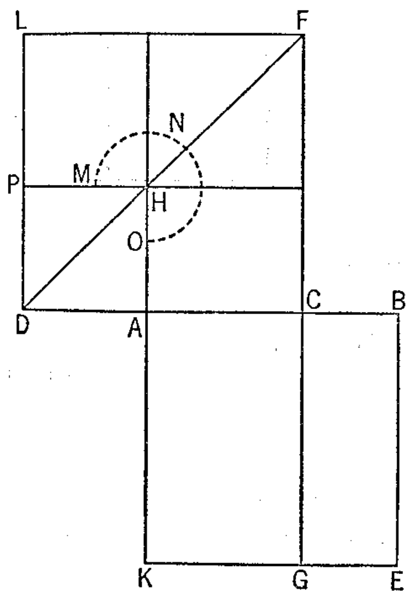
PROPOSITIONS

Proposition 1

If a straight line be cut in extreme and mean ratio, the square on the greater segment added to the half of the whole is five times the square on the half.

For let the straight line AB be cut in extreme and mean ratio at the point C, and let AC be the greater segment;

let the straight line AD be produced in a straight line with CA, and let AD be made half of AB;



I say that the square on CD is five times the square on AD.

For let the squares AE, DF be described on AB, DC,

and let the figure in DF be drawn;

let FC be carried through to G.

Now, since AB has been cut in extreme and mean ratio at C,

therefore the rectangle AB, BC is equal to the square on AC. [VI. Def. 3, VI. 17]

And CE is the rectangle AB, BC, and FH the square on AC;

therefore CE is equal to FH.

And, since BA is double of AD,

while BA is equal to KA, and AD to AH,

therefore KA is also double of AH.

But, as KA is to AH, so is CK to CH; [VI. 1]

therefore CK is double of CH.

But LH, HC are also double of CH.

Therefore KC is equal to LH, HC.

But CE was also proved equal to HF;

therefore the whole square AE is equal to the gnomon MNO.

And, since BA is double of AD,

the square on BA is quadruple of the square on AD,

that is, AE is quadruple of DH.

But AE is equal to the gnomon MNO;

therefore the gnomon MNO is also quadruple of AP;

therefore the whole DF is five times AP.

And DF is the square on DC, and AP the square on DA;

therefore the square on CD is five times the square on DA.

Therefore etc. Q. E. D.

PROPOSITION 2

If the square on a straight line be five times the square on a segment of it, then, when the double of the said segment is cut in extreme and mean ratio, the greater segment is the remaining part of the original straight line.

For let the square on the straight line AB be five times the square on the segment AC of it,

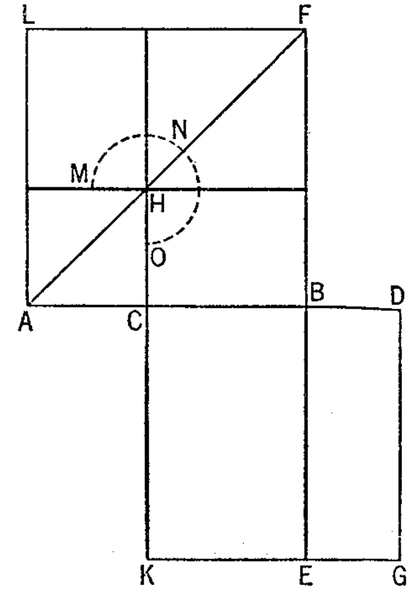
and let CD be double of AC;

I say that, when CD is cut in extreme and mean ratio, the greater segment is CB.

Let the squares AF, CG be described on AB, CD respectively,

let the figure in AF be drawn,

and let BE be drawn through.



Now, since the square on BA is five times the square on AC,

AF is five times AH.

Therefore the gnomon MNO is quadruple of AH.

And, since DC is double of CA, therefore the square on DC is quadruple of the square on CA, that is, CG is quadruple of AH.

But the gnomon MNO was also proved quadruple of AH;

therefore the gnomon MNO is equal to CG.

And, since DC is double of CA,

while DC is equal to CK, and AC to CH,

therefore KB is also double of BH. [VI. 1]

But LH, HB are also double of HB;

therefore KB is equal to LH, HB.

But the whole gnomon MNO was also proved equal to the whole CG;

therefore the remainder HF is equal to BG.

And BG is the rectangle CD, DB,

for CD is equal to DG;

and HF is the square on CB;

therefore the rectangle CD, DB is equal to the square on CB.

Therefore, as DC is to CB, so is CB to BD.

But DC is greater than CB;

therefore CB is also greater than BD.

Therefore, when the straight line CD is cut in extreme and mean ratio, CB is the greater segment.

Therefore etc. Q. E. D.

LEMMA

That the double of AC is greater than BC is to be proved thus.

If not, let BC be, if possible, double of CA.

Therefore the square on BC is quadruple of the square on CA;

therefore the squares on BC, CA are five times the square on CA.

But, by hypothesis, the square on BA is also five times the square on CA;

therefore the square on BA is equal to the squares on BC, CA:
which is impossible. [II. 4]

Therefore CB is not double of AC.

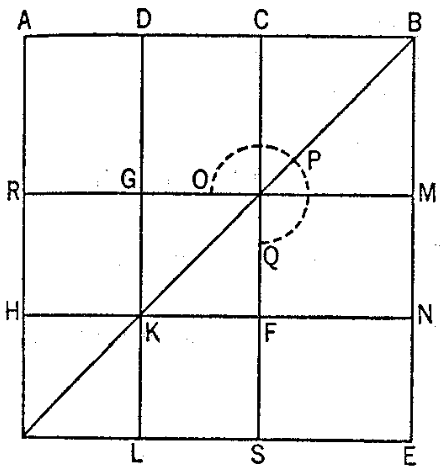
Similarly we can prove that neither is a straight line less than CB double of CA;
for the absurdity is much greater.

Therefore the double of AC is greater than CB. Q. E. D.

Proposition 3

If a straight line be cut in extreme and mean ratio, the square on the lesser segment added to the half of the greater segment is five times the square on the half of the greater segment.

For let any straight line AB be cut in extreme and mean ratio at the point C,
let AC be the greater segment,
and let AC be bisected at D;



I say that the square on BD is five times the square on DC.

For let the square AE be described on AB,
and let the figure be drawn double.

Since AC is double of DC,
therefore the square on AC is quadruple of the square on DC,
that is, RS is quadruple of FG.

And, since the rectangle AB, BC is equal to the square on AC,
and CE is the rectangle AB, BC,
therefore CE is equal to RS.

But RS is quadruple of FG;
therefore CE is also quadruple of FG.

Again, since AD is equal to DC,
HK is also equal to KF.

Hence the square GF is also equal to the square HL.

Therefore GK is equal to KL, that is MN to NE;
hence MF is also equal to FE.

But MF is equal to CG;
therefore CG is also equal to FE.

Let CN be added to each;
therefore the gnomon OPQ is equal to CE.

But CE was proved quadruple of GF;
therefore the gnomon OPQ is also quadruple of the square FG.

Therefore the gnomon OPQ and the square FG are five times FG.

But the gnomon OPQ and the square FG are the square DN.

And DN is the square on DB, and GF the square on DC.

Therefore the square on DB is five times the square on DC. Q. E. D.

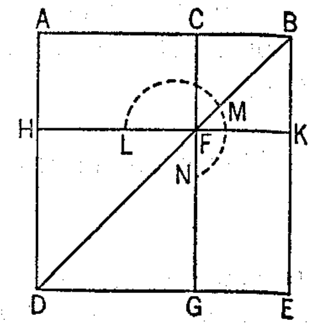
Proposition 4

If a straight line be cut in extreme and mean ratio, the square on the whole and the square on the lesser segment together are triple of the square on the greater segment.

Let AB be a straight line,

let it be cut in extreme and mean ratio at C, and let AC be the greater segment;

I say that the squares on AB, BC are triple of the square on CA.



For let the square $ADEB$ be described on AB, and let the figure be drawn.

Since, then, AB has been cut in extreme and mean ratio at C,

and AC is the greater segment,

therefore the rectangle AB, BC is equal to the square on AC. [VI. Def. 3, VI. 17]

And AK is the rectangle AB, BC, and HG the square on AC;

therefore AK is equal to HG.

And, since AF is equal to FE,

let CK be added to each;

therefore the whole AK is equal to the whole CE;

therefore AK, CE are double of AK.

But AK, CE are the gnomon LMN and the square CK;

therefore the gnomon LMN and the square CK are double of AK.

But, further, AK was also proved equal to HG;

therefore the gnomon LMN and the squares CK, HG are triple of the square HG.

And the gnomon LMN and the squares CK, HG are the whole square AE and CK, which are the squares on AB, BC,

while HG is the square on AC.

Therefore the squares on AB, BC are triple of the square on AC.

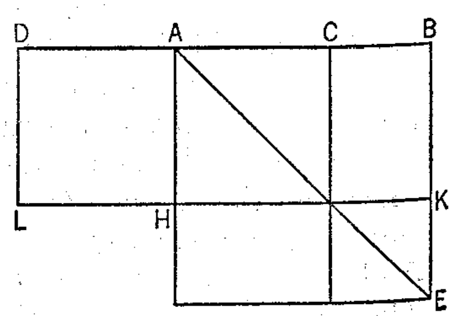
Q. E. D.

Proposition 5

If a straight line be cut in extreme and mean ratio, and there be added to it a straight line equal to the greater segment, the whole straight line has been cut in extreme and mean ratio, and the original straight line is the greater segment.

For let the straight line AB be cut in extreme and mean ratio at the point C, let AC be the greater segment, and let AD be equal to AC.

I say that the straight line DB has been cut in extreme and mean ratio at A, and the original straight line AB is the greater segment.



For let the square AE be described on AB, and let the figure be drawn.

Since AB has been cut in extreme and mean ratio at C,

therefore the rectangle AB, BC is equal to the square on AC. [VI. Def. 3, VI. 17]

And CE is the rectangle AB, BC, and CH the square on AC;

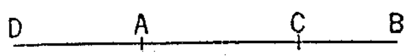
therefore CE is equal to HC.

But HE is equal to CE,

and DH is equal to HC;
therefore DH is also equal to HE.
Therefore the whole DK is equal to the whole AE.
And DK is the rectangle BD, DA,
for AD is equal to DL;
and AE is the square on AB;
therefore the rectangle BD, DA is equal to the square on AB.
Therefore, as DB is to BA, so is BA to AD. [vi. 17]
And DB is greater than BA;
therefore BA is also greater than AD. [v. 14]
Therefore DB has been cut in extreme and mean ratio at A, and AB is the greater segment. Q. E. D.

Proposition 6

If a rational straight line be cut in extreme and mean ratio, each of the segments is the irrational straight line called apotome.

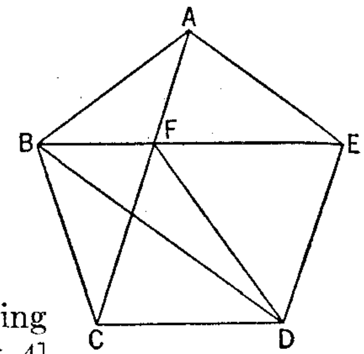


Let AB be a rational straight line,
let it be cut in extreme and mean ratio at C,
and let AC be the greater segment;
I say that each of the straight lines AC, CB is the irrational straight line called apotome.
For let BA be produced, and let AD be made half of BA.
Since, then, the straight line AB has been cut in extreme and mean ratio,
and to the greater segment AC is added AD which is half of AB,
therefore the square on CD is five times the square on DA. [xiii. 1]
Therefore the square on CD has to the square on DA the ratio which a number has to a number;
therefore the square on CD is commensurable with the square on DA. [x. 6]
But the square on DA is rational,
for DA is rational, being half of AB which is rational;
therefore the square on CD is also rational; [x. Def. 4]
therefore CD is also rational.
And, since the square on CD has not to the square on DA the ratio which a square number has to a square number,
therefore CD is incommensurable in length with DA; [x. 9]
therefore CD, DA are rational straight lines commensurable in square only;
therefore AC is an apotome. [x. 73]
Again, since AB has been cut in extreme and mean ratio,
and AC is the greater segment,
therefore the rectangle AB, BC is equal to the square on AC. [vi. Def. 3, vi. 17]
Therefore the square on the apotome AC, if applied to the rational straight line AB, produces BC as breadth.
But the square on an apotome, if applied to a rational straight line, produces as breadth a first apotome; [x. 97]
therefore CB is a first apotome.
And CA was also proved to be an apotome.
Therefore etc. Q. E. D.

PROPOSITION 7

If three angles of an equilateral pentagon, taken either in order or not in order, be equal, the pentagon will be equiangular.

For in the equilateral pentagon *ABCDE* let, first, three angles taken in order, those at *A*, *B*, *C*, be equal to one another;



I say that the pentagon *ABCDE* is equiangular.

For let *AC*, *BE*, *FD* be joined.

Now, since the two sides *CB*, *BA* are equal to the two sides *BA*, *AE* respectively,

and the angle *CBA* is equal to the angle *BAE*,

therefore the base *AC* is equal to the base *BE*,

the triangle *ABC* is equal to the triangle *ABE*,

and the remaining angles will be equal to the remaining angles, namely those which the equal sides subtend, [I. 4]

that is, the angle *BCA* to the angle *BEA*, and the angle *ABE* to the angle *CAB*;

hence the side *AF* is also equal to the side *BF*. [I. 6]

But the whole *AC* was also proved equal to the whole *BE*;

therefore the remainder *FC* is also equal to the remainder *FE*.

But *CD* is also equal to *DE*.

Therefore the two sides *FC*, *CD* are equal to the two sides *FE*, *ED*;

and the base *FD* is common to them;

therefore the angle *FCD* is equal to the angle *FED*. [I. 8]

But the angle *BCA* was also proved equal to the angle *AEB*;

therefore the whole angle *BCD* is also equal to the whole angle *AED*.

But, by hypothesis, the angle *BCD* is equal to the angles at *A*, *B*;

therefore the angle *AED* is also equal to the angles at *A*, *B*.

Similarly we can prove that the angle *CDE* is also equal to the angles at *A*, *B*, *C*;

therefore the pentagon *ABCDE* is equiangular.

Next, let the given equal angles not be angles taken in order, but let the angles at the points *A*, *C*, *D* be equal;

I say that in this case too the pentagon *ABCDE* is equiangular.

For let *BD* be joined.

Then, since the two sides *BA*, *AE* are equal to the two sides *BC*, *CD*,

and they contain equal angles,

therefore the base *BE* is equal to the base *BD*,

the triangle *ABE* is equal to the triangle *BCD*,

and the remaining angles will be equal to the remaining angles,

namely those which the equal sides subtend; [I. 4]

therefore the angle *AEB* is equal to the angle *CDB*.

But the angle *BED* is also equal to the angle *BDE*,

since the side *BE* is also equal to the side *BD*. [I. 5]

Therefore the whole angle *AED* is equal to the whole angle *CDE*.

But the angle *CDE* is, by hypothesis, equal to the angles at *A*, *C*;

therefore the angle *AED* is also equal to the angles at *A*, *C*.

For the same reason

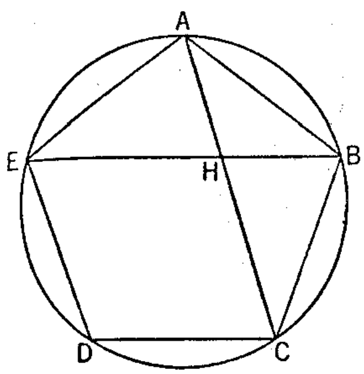
the angle *ABC* is also equal to the angles at *A*, *C*, *D*.

Therefore the pentagon *ABCDE* is equiangular. Q. E. D.

PROPOSITION 8

If in an equilateral and equiangular pentagon straight lines subtend two angles taken in order, they cut one another in extreme and mean ratio, and their greater segments are equal to the side of the pentagon.

For in the equilateral and equiangular pentagon $ABCDE$ let the straight lines AC, BE, cutting one another at the point H, subtend two angles taken in order, the angles at A, B;



I say that each of them has been cut in extreme and mean ratio at the point H, and their greater segments are equal to the side of the pentagon.

For let the circle $ABCDE$ be circumscribed about the pentagon $ABCDE$. [IV. 14]

Then, since the two straight lines EA, AB are equal to the two AB, BC,
and they contain equal angles,
therefore the base BE is equal to the base AC,
the triangle ABE is equal to the triangle ABC,
and the remaining angles will be equal to the remaining angles respectively, namely those which the equal sides subtend. [I. 4]

Therefore the angle BAC is equal to the angle ABE;
therefore the angle AHE is double of the angle BAH. [I. 32]

But the angle EAC is also double of the angle BAC,
inasmuch as the circumference EDC is also double of the circumference CB; [III. 28, VI. 33]
therefore the angle HAE is equal to the angle AHE;
hence the straight line HE is also equal to EA, that is, to AB. [I. 6]

And, since the straight line BA is equal to AE,
the angle ABE is also equal to the angle AEB. [I. 5]

But the angle ABE was proved equal to the angle BAH;
therefore the angle BEA is also equal to the angle BAH.

And the angle ABE is common to the two triangles ABE and ABH;
therefore the remaining angle BAE is equal to the remaining angle AHB; [I. 32]
therefore the triangle ABE is equiangular with the triangle ABH;
therefore, proportionally, as EB is to BA, so is AB to BH. [VI. 4]

But BA is equal to EH;
therefore, as BE is to EH, so is EH to HB.

And BE is greater than EH;
therefore EH is also greater than HB. [V. 14]

Therefore BE has been cut in extreme and mean ratio at H, and the greater segment HE is equal to the side of the pentagon.

Similarly we can prove that AC has also been cut in extreme and mean ratio at H, and its greater segment CH is equal to the side of the pentagon. Q. E. D.

PROPOSITION 9

If the side of the hexagon and that of the decagon inscribed in the same circle be added together, the whole straight line has been cut in extreme and mean ratio, and its greater segment is the side of the hexagon.

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