CHAPTER VI

HYDRAULICS AFTER FRONTINUS' TIME

Nil sine magno
Vita labore dedit mortalibus.
HORACE (65—8 B. C.),
Satires, i. 9, 59.

Life has given nothing to mortals without great labor.

TO appreciate Frontinus' position with regard to a proper knowledge of the velocity of efflux, and generally of the velocity of running water, it is instructive to follow the development of the art from his time until we arrive at the formula $v = \sqrt{2gh}$, now known to every beginner in hydraulic science, and the very foundation-stone of that science as it is known at the present day.

This formula, and the numerical values it gives to velocities of efflux, was not discovered until about the year 1738, when Daniel Bernouilli and John Bernouilli, his father, each published a different mathematical demonstration of this law. It thus appears that Frontinus wrote some 1640 years before this fundamental fact was known. By mere number of years in anticipation, he was therefore as much in the dark respecting numerical values for velocity of efflux, as we are concerning the latest discovery in hydraulics that will have appeared in the year 3538; and even by making due allowance for the greater rate of speed with which discoveries are now made, compared with ancient times, he was as far in anticipation of the year 1738, as we are of the year 2300, or 2400. "Better fifty years of Europe," says the poet, "than a cycle of Cathay," and Frontinus' time just preceded a cycle of stagnation, and even of retrogression, compared to which Cathay may be said to progress with reasonable celerity.

During 1200 years after Frontinus, practically no progress was made in the arts and sciences. The first awakening to a new life, to a revival of learning, may be dated from Roger Bacon, 1214–1293, who preached the importance of experiment, and declared knowledge in his day to be but in its infancy. We who have been educated in English-speaking countries have been accustomed to consider Lord Francis Bacon, 350 years later (1561–1626), as the author and apostle of the experimental method of studying science. But he himself made no experiments of any note, and modern research shows him to be entitled to the latter credit, only as he influenced his countrymen of Great Britain. Because a hundred years before his time lived that remarkable painter, sculptor, teacher, and engineer, Leonardo da Vinci[1] (1452–1519), the misfortune of whose fame it has been that his voluminous works, hidden away for centuries in private keeping, and exposed to manifold vicissitudes, found no publisher until the last few years; and have, even to-day, not been before the public long enough to be used by modern writers as they undoubtedly will be. He not only preached the duty of study by means of experiment, but was himself a most prolific experimenter, and a teacher. In the last named way, he anticipates Lord Bacon; in the other, he is the forerunner even of Galileo. This is what he says on the first-named subject:—

"In the examination of physical problems I begin by making a few experiments, because it is my desire to state the problem, after I have had the experience of it, and then to show why it is that the bodies are forced to act in the described manner. This is the method it is necessary to follow in all examinations concerning the phenomena of nature. It is true that nature begins, as it were, with argument and ends with experience, but nevertheless we must follow the contrary way; as I have said, we must commence with experience, and strive by means of it to discover truth.

"In the study of the sciences which are allied to mathematics, those who do not consult nature, or authors who are not the pupils of nature, are merely little children. I say it emphatically. Nature alone is the true teacher of true ability. And yet, behold the stupidity of it! The world makes merry over a man who prefers to learn from nature rather than from writers who themselves could only be the pupils of nature."

[1] Grothe, Hermann, *L. da Vinci als Ingenieur*, etc., Berlin, 1874.

His experiment on the law of falling bodies is most interesting in connection with the matter we are now considering.[1] He used two long boards hinged together like the leaves of a book. On the inside these boards were smeared with tar or wax. A string latch served to close them suddenly. He then takes a small tube, filled with shot, the tube having nearly the same diameter as the shot. This tube is held vertically in and over the angle of the wooden book, itself set up vertically. The shot are then allowed to drop out, and on pulling the latch are caught, as they fall between the leaves of the wooden book; and their relative distances, as they are falling, are impressed on the tar or wax covering of the boards.

Until quite recently Galileo has been supposed to have been the first experimenter on the laws of falling bodies, but here was this great engineer and teacher busily at work at it one hundred years previously. However, with Galileo (1564–1642), we first touch the modern science of "dynamics," or of bodies in motion. Says Rühlman:[2] "For the proper founding of the science of dynamics, or of the science which treats of the causes and the laws of motion, were requisite talents of a degree of eminence, such as the Lord Almighty called into being with Galileo in the year 1564."[3] But Galileo had no proper means for measuring time, no clocks or watches. Both he and his son tried to make a clock but did not succeed. Instead, he used a large bowl of water, having a small orifice at the bottom, and compared times by the weights of water discharging during these times; using his finger to start and stop the flow of water out of the bowl. As we shall see, it is a reasonable assumption that this makeshift of a clock became, in the hands of Galileo's pupils, and of those of his pupil's pupil, the suggestion for an experimental demonstration of the laws of efflux in general.

Castelli (1577–1644), the pupil of Galileo, was a Benedictine monk, from that same Montecassino which saved Frontinus' commentary to posterity, and he first showed that the quantity of efflux, in a given

[1] Da Vinci, Leonardo, *Del Moto e Misura dell Acqua*, Bologna, 1828, p. 364.

[2] *Gesch. d. technischen Mechanik*, p. 53.

[3] Libri, *Histoire des Sciences Mathématiques*, iv. 160, 466, gives proof positive that Lord Francis Bacon knew Galileo's work, published and unpublished, a year before the publication of the *Novum Organon*. See, also, Hume, *History of Great Britain*, 1770, iv. 215.

time, depended by law on, or was a function of, the depth of water in a bowl such as the one just spoken of; that is, was a function of the head. But he wrongly stated this law, making the quantity vary directly as the head. It was his pupil, Torricelli (1608–1647), the inventor of the barometer, the grandson, in a professional sense, of Galileo, who first proved, in 1644, or only two years after Galileo's death, that the velocities of efflux are as the square roots of the head. But this still furnished no numerical value for the velocity of efflux. Still another, and still other great men, had to devote their lives to this cause; thirty more years had to pass by, till Huygens (1629–1695), the inventor of pendulum clocks, first found the numerical value of the acceleration of gravity, commonly represented by the letter g, in 1673; and sixty-five more years had to elapse, until the genius of the two Bernouillis, father and son, in 1738, or two hundred and fifty years after Leonardo da Vinci, finally laid the foundation of modern determinate hydraulics, by writing the equation of $v = \sqrt{2gh}$, every letter and character of which may be considered the contribution of, and a tribute to, the skill and perseverance of one or more of the many great men that have been named. v may stand to symbolize the experiments of da Vinci and of Galileo, and the preaching of the two Bacons; $2g$ alone would suffice to immortalize Huygens, were he not already permanently distinguished by his invention of pendulum clocks and other works; h may serve to recall Castelli; and the square root sign, his pupil, Torricelli; and when next we write the formula, let us remember that it took two hundred and fifty years of work, not to speak of another and a preceding two hundred and fifty years or more of speculation, to put it upon the blackboard of the world. But no amount of speculation alone, or of peripatetic philosophy, would have produced it. To do that, the work of centuries of earnest men, not too proud to dip their hands into bowls of water, and to *experiment* in hydraulics, the while they were wearing mechanic's overalls, so to speak, was absolutely necessary.

No wonder then that Frontinus, who lived in an age of peripatetic philosophy, did no better than we have found in his two books, in the way of gauging the water supply of ancient Rome.

CHAPTER VII

ARITHMETIC, A. D. 97, AMONG THE ROMANS

. . . 'Dicat
Filius Albini, si de quincunce remota est
Uncia, quid superat? — Poteras dixisse.'
'Triens.' — 'Eu!
Rem poteris servare tuam! — Redit uncia; quid fit?'
'Semis.'

HORACE (65-8 B. C.), *Ars Poetica*, 325.

'Let Albinus' boy say, if from a five-twelfth an ounce be taken, what will remain? You know.' 'A third.' 'Very good! You'll be able to take care of your property! If an ounce be added, what will it make?' 'A half.'

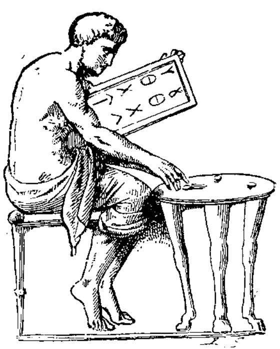
ONE of the astonishing things noted in reading the book of Frontinus is the readiness with which he performs his arithmetical computations, without knowing anything of the Arabic or Indian system of notation, or of common or decimal fractions. To him fractions are entities having names, and represented by hieroglyphics. They have certain relations to unity, and to one another, indeed, but just how he keeps track of them all, and manages to use the lot, is as great a puzzle as the Chinese alphabet, which his set of fractions in some respects resembles.

After the first four of the larger fractions, they are all of the duodecimal order, the list being as follows: $1/2$, $1/3$, $1/4$, $1/6$, $1/12$, $1/24$, $1/36$, $1/48$, $1/72$, $1/288$, with a name for each one. (Semis, Triens, Quadrans, Sextans, Uncia, Semuncia, Duella, Sicilicus, Sextula, Scripulus.) There are also symbols and names for $5/12$, $7/12$, $2/3$, $3/4$, $10/12$, and $11/12$. The division by twelve dates back, as is known, to a period of great antiquity, and has come down to us in the twenty-four hours of the day, and in the sixtieths of the hour, and the sixtieths of that, as used, together with duodecimals, by the Greeks; who, according to Herodotus, learnt them of the Babylonians,[1] the sixtieths being the modern minutes and seconds,

[1] Herodotus, ii. 109.

originally called "*partes minutae primae*" and "*partes minutae secundae*."[1]

The relation of the circumference and the area of the diameter of a circle is given by Frontinus with the use of $\pi =$ as near $3\frac{1}{7}$, $=$ about 3.1429, as the use of the clumsy fractions, just named, selected and added up, will permit of. Working with duodecimal fractions, it would have been far easier to take $\pi = 3\frac{1}{8}$, as had been done by Vitruvius,[2] only one hundred years before Frontinus. But he laboriously ploughs along with $\pi = 3\frac{1}{7}$, and is always pretty nearly right: — except when he, or some copyist, or translator, or printer, has made a mistake; which is not without example.



CALCULATOR.[3]

Thus, he tells us, the *quinaria* has an interior diameter of one digit and three unciae; or as we should say: 1.25 digits, the digit being $\frac{1}{16}$ of a foot; "in circumference, three digits, a half, five unciae and three scripuli;" or, as we would say: 3.92709 + digits. Calculated on the basis of $\pi = 3\frac{1}{7}$, this figure would be 3.9285+, and for the ordinarily used value of $\pi = 3.1416$, it is 3.9270.

His areas are $\frac{\pi d_1^2}{4}$ and he well knows that the areas are as the squares of the diameters. But when it comes to the conception of a cubic foot, he seems to avoid it, and appears to be in total ignorance of so much as a conception of the idea of a procession of such cubic feet passing a given point in a unit of time; or of what we ordinarily call cubic feet per second; in which respect he is equalled, however, as we have seen, by many men, some of them of considerable standing in the community, who are living at the present day.

Frontinus ascribes the name *quinaria* to the diameter of the finished pipe, being five quarter digits in diameter;[4] and he goes on call-

[1] Cantor, *Gesch. d. Math.* i. 445.

[2] *Agrimensoren*, 88.

[3] The ancients counted by means of small stones (*calculi*). The mathematician represented here, from a gem in the *Cabinet de France*, No. 1,858 of the Chabouillet Catalogue, arranged the *calculi*, while the reckoning tablet, covered with Etruscan characters, is in his left hand. Daremberg et Saglio, *Dict. des Antiq. grecq. et rom.*, under the word *Abacus*.

[4] *De Aquis*, 25.

ing a pipe of six quarter digits in diameter, a *sextaria;* and so on, only that at twenty quarter digits in diameter it becomes nearly the same, whether the name *vicenaria* be ascribed to quarter digits diameter, or to square digits of area. From this size upwards he names the pipes by their areas in square digits, stopping at a pipe of one hundred and twenty square digits area, nearly nine inches in diameter.[1]

[1] Pliny (xxxi. 31) and Vitruvius (viii. 6, 4) name the pipes by their circumferences, instead of by their diameters, as is now customary. Rope is sold by circumference measure at the present day.

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